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Arts and sciences: Citizens of parallel universe?

Artes y ciencias: ¿Ciudadanas de universos paralelos?

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Resumen

A pesar de la existencia documentada de individualidades históricas capaces de integrar artes y ciencias, las instituciones occidentales contemporáneas encargadas de formar a artistas y científicos son claro reflejo del paradigma social que los presenta como agua y aceite. La deconstrucción algebraica de una imagen de la serie *Universos Yin* muestra cómo en ella convergen y se complementan matemáticas, computación simbólico-gráfica, taoísmo y arte para representar un concepto abstracto. A partir de esto se hacen algunas propuestas académicas integradoras, que podrían coadyuvar a transformar las dualidades disjuntas actuales en dualidades sinérgicas complementarias.

Palabras claves: Artes y ciencias, artes visuales, matemáticas, computación simbólico-gráfica, taoísmo.

Abstract

Despite the documented existence of historical personalities who could integrate arts and sciences, contemporary western institutions in charge of the education of new artists and scientists clearly reflect the extended social paradigm that represents them like water and oil. The algebraic deconstruction of an image belonging to the *Yin Universes* series shows how mathematics, symbolic-graphical computation, Taoism and art converge and complement each other to represent an abstract concept. From this, some integrating academic proposals are formulated, that could help to transform present disjoint dualities into complementary synergic dualities.

Keywords: Arts and sciences, visual arts, mathematics, symbolic-graphic computation, Taoism.

1 Introduction

If in a first and very reductionist approach, one thinks of the arts and the sciences, not as abstract ideas and concepts with precise intrinsic epistemological meanings, but pragmatically as the set of activities artists and scientists develop, everything would make us think that the arts and the sciences live and evolve in parallel universes, inhabited by artists and scientists, respectively, that by education ignore each other, profess disjoint bodies of knowledge, and lack common languages allowing a smooth and fluid communication between them. This landscape does not substantially vary if we characterize the arts and the sciences through their products, created for, and oriented to, satisfying needs belonging to two sub-universes, also essentially disjoint and complementary: the sub-universe of the spiritual, aesthetic, and emotional activities and needs, in the case of the arts, and the sub-universe of the material, objective, and intellectual needs, in the case of the sciences. If we examine the role technology plays in the arts and the sciences, the conclusion is not too much different: artists are users of technological tools whose foundations they ignore, while scientists are trained to create and dominate the foundations of technologies, the vast majority of whose applications they ignore.

Are really the arts and the sciences citizens of parallel universes? Or does the disjoint duality between arts and sciences speak more about western minds and cultures, than about the arts and the sciences themselves? The existence of well-documented cases of scientists with serious artistic interests, and artists with strong interests in mathematics, physics, and technology, since the most remote antiquity to the contemporaneity, seem to speak more about the second than the first hypothesis. The great importance of historical individualities notwithstanding, a first preliminary review of the academic program of studies in the fields of sciences, engineering, and arts in the most important Venezuelan universities (UCV 2017, ULA 2016, USB 2011) shows that the intersection between the studies of the arts and the sciences is nearly empty. Even though this first review should be extended and deepened to improve its scientific support, the first impressions seem to indicate that the situation is basically the same in other Western countries.

Even though the available space does not permit to go deeper into details, it is worth to mention that disjoint duality serves as the topological and geometrical foundation of many models of organizations, both physical and theoretical

(Rodríguez-Millán 2019). Such models may even be the optimal ones to explain the behavior of relatively simple low-order systems, but may be of limited application if one pretends to use them to model and understand the dynamics of uncertain, interconnected, high-order, time-varying, complex, systems.

If present disjoin duality were recognized as an obstacle to the future development of art and science, would it be possible to do something in this respect? How could we help present aspirants to artists and scientists becoming better prepared and equipped to afford the exercise of their artistic and scientific disciplines, let us say, 10 years ahead?

2 Mathematics: Geometry vs. Algebra or Explaining vs. Computing

The pieces of work considered in this essay are visual metaphors, technically multidisciplinary and interdisciplinary in nature, that were constructed using integrated symbolical-graphical-numerical computational procedures, with which the author pretends to explore and represent complex, yet concrete, concepts like those of monoculturality, multiculturality, interculturality, and transculturality, if we study them within the global context of human interactions, or the topologically equivalent concepts of monodisciplinarity, multidisciplinarity, interdisciplinarity, and transdisciplinarity, if we pose them within the local level of the university academic life. Mathematically speaking, these oeuvres may be thought of as images of discrete-time (Ogata 1995) stochastic dynamical systems (Hirsch-Smale 1974, Arnol'd 1974), that is to say, systems that are only defined at some particular instants of time, and present some uncertain or unpredictable features of behavior.

Each discipline has its own tradition, its own local culture, its own collective myth, that at the end finish determining what it is written and how, what is it said, and what it is left in the shadows. Mathematics is an elegant and mysterious lady, always very much concerned about her public image and extraordinarily cautious about her private life, as a consequence of which she never reveals the details of her creative processes, convinced perhaps that the creative process of each mathematician is unique and personal, and therefore that that which is the basis and support of the intuition of one of them is not transferable to any other one. Very much against mathematical writing traditions, in the following sections, we will deconstruct an oeuvre to explicitly reveal its basic algebraic structure, and the part of the oeuvre that can be algorithmized and transformed into a Mathematica code, to be afterwards implemented using a digital computer, exported as a .jpg image, and finally printed, just as if it were a standard photographic image.

The deconstruction we will perform next will also clearly show something every good painter or photographer knows well, namely, that dominating the technique of using a brush, or owning the last high-tech camera, does not automatically transform the owner of the brush into a good painter, or the owner of the camera into a good photographer. Fortunately, the arts, like the sciences, transcend their instruments and techniques.

3 A Yin Universe

A Yin Universe, the image shown in Figure 1, belongs to a sequence of images created along the process of studying, developing, and constructing a *Taijitu*, the well-known universal symbol representing the Taoist concept of Yin-Yang (Wang 2010, Bielba and Zabaleta 2006), out of yin colors, concepts, and forms: different tones of blue, even numbers, and even regular polygons, respectively.

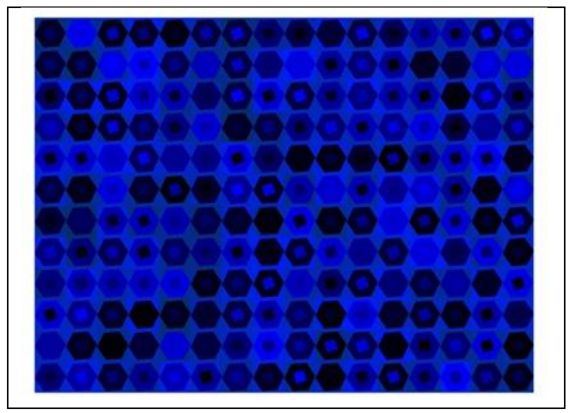

Figure 1. *A Yin Universe*. *A Yin Universe* may be algebraically thought of as a sequence of cells, which are run one by one, from left to right, from bottom to top, at equal intervals of time T. This algorithmic course implicitly fixes the origin of coordinates at the lower-left extreme point of the image. Without losing generality we suppose $T = 1$.

Perhaps the easiest way to introduce the concept of a *discrete-time dynamical system* (Arnol'd 1974, Hirsch and Smale 1974, Ogata 1995) to a visual artist is through a table collecting the "coordinates" of the studied system at each instant of time. If we assimilate the oeuvre *A Yin Universe* to a regular uniform mosaic, a familiar object to anyone who attended a general course on the history of western art, we could see that it consists of 192 square tesserae of identical dimensions. Identical dimensions do not mean, however, that they are freely interchangeable, and therefore in order to univocally describe *A Yin Universe* in terms of its constituent tesserae, we must solve two different and independent problems: to univocally describe the content of each tessera, and to unequivocally indicate its position.

3.1 Locating the tesserae on A Yin Universe

Unequivocally locating each tessera in *A Yin Universe* is a sequentialization problem that we will solve identifying the (horizontal) rows and the (vertical) columns of the mosaic through the counters "i" and "j", respectively. So, $i = 1$ corresponds to the lowermost or first row, $i = 2$ to the second row from bottom up, and so on. Accordingly, $j = 1$ denotes the first column from left to right, $j = 2$ is the second column from left to right, and so on. The introduction of a counting system serves the dual purpose of counting and ordering the sets of rows and columns, which derivatively also defines a system of coordinates that permits locating and identifying any tessera with absolute precision. Thus, the (i, j)~tessera is the tessera belonging to the i~*row* and the j~*column* of the mosaic, which is now univocally defined and localize.

A Yin Universe consists of 12 rows and 16 columns; then $1 \leq i \leq 12$, $1 \leq j \leq 16$, and the whole mosaic contains 192 (i, j)~tesserae. The sequentialization principle of the tesserae was described in the caption of Figure 1: from left to right, from bottom to top, at equal intervals of time T. According to these conventions, the sequence of tesserae in *A Yin Universe* is:

Row	(i, j)~tesserae	Continous Sequencing
1	$\{\{1,1\},\{1,2\},\dots,\{1,16\}\}$	$\{1,2,\dots,16\}$,
2	$\{\{2,1\},\{2,2\},\dots,\{2,16\}\}$	$\{17,18,\dots,32\}$
…	………..	………
12	$\{\{12,1\},\{12,2\},\dots,\{12,16\}\}$	$\{177,\dots,192\}$

The continuous sequentialization of the tesserae of *A Yin Universe* exposed above describes the way the whole image was originally coded and created in Mathematica. Yet, sequentialization admits manifold and equally valid solutions and implementations: just to mention a few of them, *A Yin Universe* could have been read in western key, starting at the upper left tessera, from left to right, and from top to bottom; also in Arabic key, starting at the upper right tessera, from right to left, and from top to bottom; yet also in Japanese key, from top to bottom, and from right to left, starting at the upper right tessera; or following any other abstract reading key generated by the fantasy of the artist.

3.2 Constructing the Tesserae of A Yin Universe

3.2.1 The First Layer of the Tesserae

Once the tesserae's location problem has been solved, we may proceed to describe the construction of each tessera. Figures 2, 3, and 4 show that each tessera contains three layers, the deepest of which is a square cell of side X, colored in a tone of blue. Just like in the sequentialization problem, the procedure to construct each tessera is not necessarily unique, and choosing a particular construction recipe use to obey a multifactorial criterion: an aesthetic preference of the author, a conceptual systematization principle, a criterion of programming optimization, etc. For the sake of this essay, we chose a criterion of geometric systematization for the construction of the tesserae: we first construct a universal tessera in the lower-left corner of the mosaic, and then translate it to the arbitrary generic position (i, j), located at the intersection of the ith~column with the jth~row. In the sequel, we will reserve the blue color for the x-coordinates and the horizontal translations, and the orange color for the y-coordinates and the vertical translations.

A universal rectangle, with a vertex at the origin $(0, 0)$, base X, and height Y, can be represented, in the syntax of Mathematica (), in the following way:

$$Rectangle[\{0,0\},\{X,Y\}], \tag{1}$$

where $\{0,0\}$ represents the origin, and $\{X,Y\}$ represents the antipodal vertex on the principal diagonal of the rectangle. In order to color the rectangle, we must provide a color directive that, by default, will be assumed to be given in the RGB code:

$$RGBColor[r,g,b], \tag{2}$$

where the color coordinates $r, g, b \in [0,1] \subset \mathbb{R}$. Thus, the desired universal blue tessera located at the origin is represented by

$$RGBColor[0,0,b], Rectangle[\{0,0\},\{X,X\}], \tag{3}$$

where $b \in [0,1]$. To generate the tessellation in Figure 2:

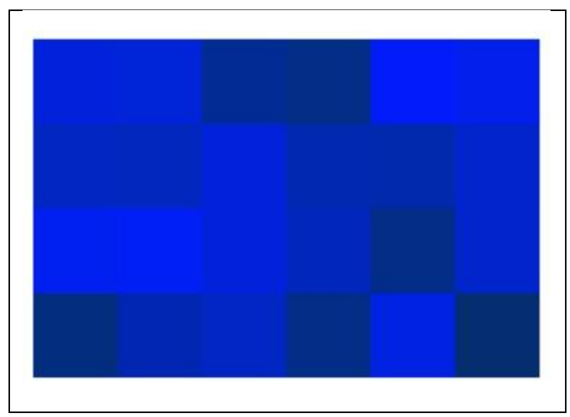

Figure 2. The first layer of *A Yin Universe*. *A Yin Universe* could be constructed using different strategies, which are visually but not mathematically or computationally equivalents. In this essay, we will assume that *A Yin Universe* was constructed in three layers. The image shows a zoom of the lower-left corner of the image, consisting of 4 rows and 6 columns.

we need to translate the universal tessera located at the origin step by step to the right and upward, to cover all possible cells of the mosaic without leaving empty spaces. The universal tessera at the origin, horizontally translated to the i-th column would have the following form:

$$RGBColor[0,0,b], Rectangle[\{0+iX,0\},\{X+iX,X\}] \tag{4}$$

where $b \in [0,1]$, and this new tessera, translated upwards along the i-th column until reaching the j-the row would be given by:

$$RGBColor[0,0,b],\ Rectangle[\{i\,X, 0 + jX\},\{(i+1)X, X + jX\}], \quad (5)$$

where $b \in [0,1]$.

We have so far constructed a universal blue square at the origin, which can be stepwise moved to any arbitrary (i, j)~position over the whole area to be covered with a mosaic. Covering an area using a single universal rectangular tessera is the simplest strategy to construct regular (disjoint) partitions of given surfaces, an important topological problem, all whose possible solutions are well known since the remote antiquity. To transform the covering strategy above into an algorithmic (then computer-programmable) construction tool we need an operative definition of a tessera, something that could also be formulated manifoldly. For a mathematician, it is natural to model objects using functions of several variables, whose images (the artistic results) can be modulated through the appropriate choice of values for variables and parameters. In the syntax of Mathematica, a tessera could be defined as follows:

$$Tessera[i\_, j\_, X\_, Y\_, b\_] := \{RGBColor[0,0,b],\ Rectangle[\{i\,X, j\,Y\}, \{(i+1)\,X, (j+1)\,Y\}]\}, \quad (6)$$

where *Tessera* is the name of the brand new defined function, $i\_, j\_$ are their variables, $X\_, Y\_$ are two deterministic parameters, and $b\_$ is a random parameter, respectively. We define Tessera in terms of two standard pre-defined functions of Mathematica: *RGBColor*, to provide a color directive, and *Rectangle* to generate the supporting rectangle of the tessera.

Once we defined the function *Tessera*, we can use its variable i to sequentialize the horizontal recruiting of tesserae to construct rows of tesserae:

$$TesseraRow[i\_, j\_, X\_, Y\_, b\_] := Table[Tessera[i, j, X, Y, b], \{i, 0, 15\}], \quad (7)$$

and analogously, we can use the variable j to sequentialize the recruiting of rows of tesserae to construct a matrix of tesserae:

$$TesseraMatrix[i\_, j\_, X\_, Y\_, b\_] := Table[Table[Tessera[i, j, X, Y, b], \{i, 0, 15\}], \{j, 0, 11\}]. \quad (8)$$

Having conceptually finished the construction of the first layer of *A Yin Universe*, we proceed to construct its second layer, with the intention of incorporating the regular polygons to the discussion.

3.2.2 The Second Layer of the Tesserae

Hexagons are magical geometric objects of great beauty, which can provide unexpected solutions to plenty of technological and mathematical problems. Nature frequently uses hexagons, for instance, in the construction of complex, compact, and stable structures, capable of supporting strong perturbations without losing their functional properties. Just because of their structural stability under great perturbations hexagons should be objects of reflection and study for artists and architects, and very especially for those ones interested in solving problems the way nature does.

Reapplying the construction philosophy we used for the first layer to the construction of the second layer, we will first design a universal hexagon at the lower-left cell of the mosaic and then will translate it to an arbitrary (i, j)~tessera. A *hexagon* is a regular polygon of six equal-length sides, whose canonical image in the real plane is univocally described by the coordinates of its six vertexes:

$$\left\{R\ Cos\left[\frac{2\,\pi\,k}{K} + \phi\right], R\ Sin\left[\frac{2\,\pi\,k}{K} + \phi\right]\right\}, \quad (9)$$

where $K = 6$, and $1 \le k \le 6$. Given that the six vertexes of the rectangle lie on the circle of radius R centered at the origin, by extension, we say that the hexagon also has *radius* R. The angle ϕ is the *initial rotation angle*, or the *phase angle*, of the hexagon. When $\phi = 0$ the hexagons are exactly in the position shown in Figure 3, clearly showing that the diameter of the hexagons in *A Yin Universe* coincides with the length of the sides of the squares the first layer of the mosaic consists of.

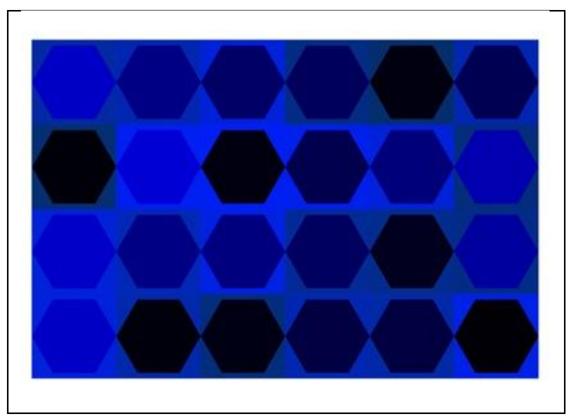

Figure 3. The second layer of *A Yin Universe* consists of hexagons, whose diameters coincide with the sides of the background square tessellation. The centers of the background squares and the hexagons also coincide.

Given that the geometric center of the hexagon, located at $\{0, 0\}$, does not coincide with the geometrical center of the lower-left square tessera of the first layer, located at the point $\{\frac{X}{2}, \frac{X}{2}\}$, it is necessary to translate the hexagon to make both geometrical centers coincide. The translated hexagon is

$$\left\{\frac{X}{2} + RCos\left[\frac{2\,\pi\,k}{K} + \phi\right], \frac{X}{2} + RSin\left[\frac{2\,\pi\,k}{K} + \phi\right]\right\} \quad (10)$$

with $K = 6$, and $1 \le k \le 6$. If we now translate this hexagon to the intersection of the ith-column with the jth-row, we obtain:

$$\left\{iX + \frac{X}{2} + R\ Cos\left[\frac{2\,\pi\,k}{K} + \phi\right],\ jX + \frac{X}{2} + R\ Sin\left[\frac{2\,\pi\,k}{K} + \phi\right]\right\}, \quad (11)$$

where $K = 6$, and $1 \le k \le 6$. To finish the construction of the second layer of the mosaic, we still need to equip the generic hexagon in position (i, j) with a color directive, and to give an operative definition of the colored translated hexagons. Let

$$Xp[i, X, \phi, k] := \left(i + \frac{1}{2}\right) X + R\ Cos\left[\frac{2\,\pi\,k}{K} + \phi\right] \quad (12)$$

$$Yp[j, Y, \phi, k] := \left(j + \frac{1}{2}\right) X + R\ Sin\left[\frac{2\,\pi\,k}{K} + \phi\right] \quad (13)$$

be the (i, j)~coordinates of the vertex of the translated hexagon, respectively. Then,

$$Table[\{Xp[i, X, \phi, k], Yp[j, Y, \phi, k]\}, \{k, 0, 6\}] \quad (14)$$

is the counterclockwise ordered list of the vertex of the (i, j)~hexagone, and

$$Polygone[Table[\{Xp[i, ..], Yp[j, ..]\}, \{k, 0, 6\}]] \quad (15)$$

is the image of the polygon. So, the function

$$ColoredHexagon[i\_, j\_, X\_, Y\_, \phi\_, b\_] := \{RGBColor[0, 0, b], Polygone[Table[\{Xp[i, ..], Yp[j, ..]\}, \{k, 0, 6\}]]\} \quad (16)$$

provides the algorithmizable definition of the (i, j)~hexagon we need to proceed to construct first rows of hexagons

$$HexagonRow[i\_, j\_, X, Y, \phi\_, b\_] := Table[ColoredHexagon[i, j, ..], \{i, 0, 15\}] \quad (17)$$

and then matrices of hexagons:

$$HexagonMatrix[i\_, j\_, X\_, Y\_, \phi\_, b\_] := Table[HexagonRow[i, j, X, Y, \phi, b], \{j, 0, 11\}]. \quad (18)$$

3.2.3 The Third Layer of the Tesserae

The construction of the third layer of *A Yin Universe*, Figure 4, is left to the reader as an exercise, which could be solved manifoldly. Among the admissible solutions, there is a very short and elegant one.

3.3 Algebra, Geometry, Art, and More

Once we have described a possible strategy, and unveiled its supporting algebra, allowing us to construct a visual piece of work like *A Yin Universe*, multiple questions naturally emerge: why did we use three-layer tesserae? Why not one, or two, or four-layer tesserae instead? Why did we choose square and hexagons? Why not square and octagons, or hexagons and octagons? How did we choose the colors? Did we use a deterministic rule for it? Or, were the colors randomly chosen? For better or worse, these are not questions of mathematical nature and therefore, mathematics has no answer for them. These are questions of aesthetic, compositional, cultural, linguistic, symbolic, philosophical, political, or religious nature that transcend the field of action of mathematics, and even more of computational sciences thought of as the set of mathematical disciplines dealing with algorithmizable, and computable in a finite number of steps, processes.

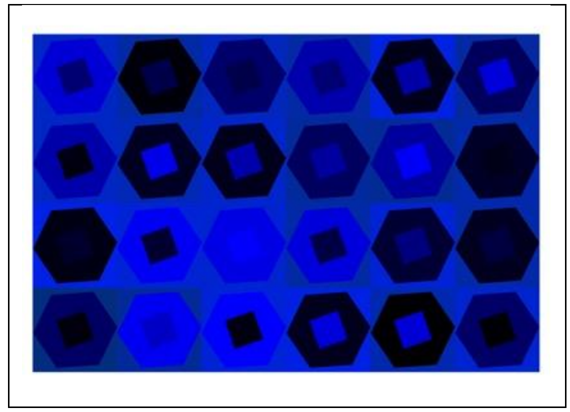

Figure 4. The third layer of A Yin Universe. The third layer of *A Yin Universe* consists of squares which are concentric with the hexagons of the second layer, and whose sides measure half the side of the squares of the first layer.

The muse is to the arts, what the intuition is to mathematics, and both types of inspirations have a lot to do with the capacity of the human mind to reach global, geometric non-algorithmizable visions of reality or, in the best case from a constructivist point of view, visions which are very difficult to tame and transform into algorithmizable and computable procedures. In this vast universe, the unique valid rule, both for the arts and the mathematics, is Pablo Picasso's haiku: "The inspiration does exist, but She must find you working", and yet, for all young artists and scientists it is worth to recall Edison's maxim: "The genius is 1% of inspiration and 99% of transpiration".

4 Arts and Dynamical Systems

What in contemporary mathematics is called dynamical systems (Arnol'd 1974, Rodríguez-Millán 2020-2) is a big and very useful chapter of mathematics dedicated to the study of systems that evolves over time. In their classic and orthodox version, the dynamical systems are manifested as differential equations (Hirsch-Smale 1974),

$$\dot{x} = f(x, u, \eta),\ \ x \in \mathbb{R}^n, u \in \mathbb{R}^n, \eta \in \mathbb{R}^k, f \in C^1, \quad (19)$$

those equations originally invented by Newton to equip mechanics with a mathematical formalism allowing formulating the laws of nature rigorously, with predicting capacity. The physicists did so well in this adventure that physics became the paradigm of successful Western science, which could predict astronomical phenomena years, centuries, and millennia in advance. In spite of their enormous success, during the transition from the nineteenth to the twentieth century, physicists and mathematician became aware that most differential equations could not be solved explicitly, as expected, what led them to search for mathematical methods permitting to study the properties of the solutions of differential equations without actually calculating them. This was the origin of the qualitative or geometric theory of differential equations, which emphasizes the study of the behavior, and very particularly the long-term behavior, of the set of trajectories described by the objects as they evolve in time over the space. In parallel to the development of the qualitative theory of differential equations also bloomed and flourished the numerical analysis, and the numerical solution of differential equations, omnipresent in contemporary daily life through digital computers.

A Yin Universe, that image as a global visual object, is a snapshot, a sample, of a visual metaphor of a monocultural, monochromatic, monophonic universe, of a One, which begins its evolution in continuous-time towards a destiny that is still waiting to be defined in full detail.

A Yin Universe, inwardly, is the story of an individual, modeled by a hexagon, whose dynamic is only defined at those discrete instants of time determined by the counters $0 \leq i \leq 15$ and $0 \leq j \leq 11$. Thus, internally, *A Yin Universe* is a discrete-time dynamical system, which only has existential reality at the instant associated to a particular sequencing of the nodes of a (i, j)~mesh, created by a poet who reads the points of the mesh according to a local cultural pattern created by his mind. The hexagon jumps from one compartment to another deterministically, but during each jump its azure monoculturality undergoes small and subtle stochastic variations that escape from any control. That mixture of absolute determinism with probabilistic uncertainty characterizes it as a discrete-time stochastic dynamical system.

5 Technical Rigidity and Conceptual Freedom

Up to this point we have developed and presented a digital algebraic technique to construct discrete-time dynamical systems, whose images might be mentally assimilated to digitally implemented abstract stochastic mathematical mosaics. In this section, we collect some pictures for an exhibition whose curatorial goal would be to give possible answers to some of the questions previously formulated in Subsection 3.3 Algebra, Geometry, Art, and More.

5.1 Why not one, or two, or four-layer tesserae?

A Yin Universe was constructed using yin symbols: blue colors, even numbers, and regular even polygons. *A Yang Universe* is also a realization of the One, yet colored in yang colors, and consisting of yang numbers and regular odd polygons. The realization of *A Yang Universe* shown in Figure 5 is a four-layer mosaic, consisting of a background square layer, and three successive layers of heptagons, pentagons, and triangles, respectively.

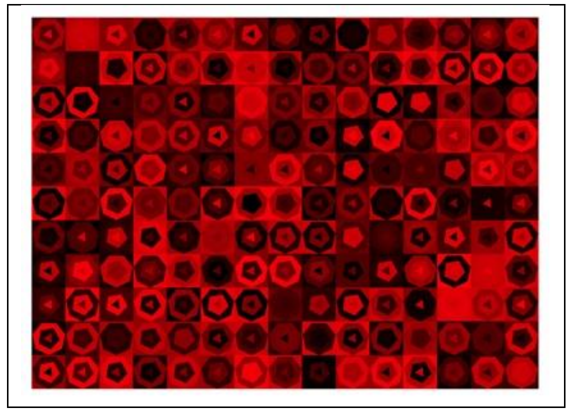

Figure 5. A Yang Universe. Yang symbols: reds, odd numbers, and regular odd polygons are used to construct a yang realization of the One.

5.2 The Brotherhood of the Blue Monks and the Tribe of the Red Knights

According to the Book of Tales of the Ancient Times (Lao Zi 2011), there first existed two isolated realizations of the One: the Brotherhood of the Blue Monks of Figure 1, and the Tribe of the Scarlet Knights of Figure 5. They grew up ignoring their mutual existences until they covered all the lands of the Ancient World. Then in the lands of both of them, strange stories of encounters with people coming from other worlds started being heard. And so there appeared the first bicultural society ever of Figure 6, as a byproduct of the encounters between the scouts of the Brotherhood of the Blue Monks, and the Tribe of the Scarlet Knights. Both the Blue Monks and the Scarlet Knights started having troubles to identify their darkest members, because sometimes the dark ones of both sides look quite similar to each other. Both sides of what? And it was so they discovered there had appeared a boundary between the Blues and the Scarlets, enforced by an infallible mathematical identification and separation mechanism (Rodríguez-Millán 2019).

Biculturality without inter-action;
Mutually tolerant coexistence in
multidimensional spaces and times;
Preservation of traditions and knowledge
from past worlds and civilizations;
No emergent aromas stemming from
fusions of condiments and spices;
No new polychromatic visions;
No new experimental polyphonic tunes;
Preservation without innovation.

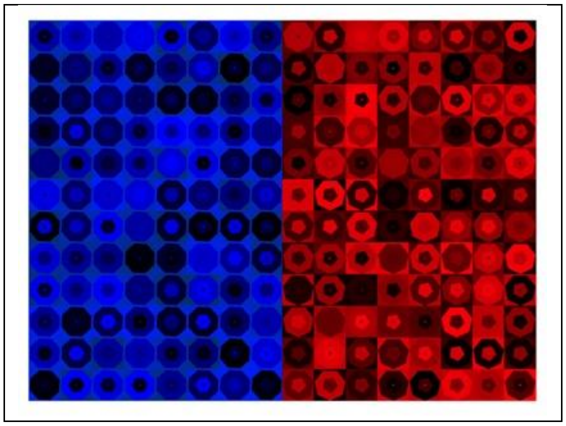

Figure 6. The First Bicultural Society. The first bicultural society we have news about, according to the Book of the Tales of the Ancient Times, consisted of the Brotherhood of the Blue Monks and the Tribe of the Red Knights.

5.3 Colors, Symbols, and Meanings

The choice of blue and red colors in *A Yin Universe* and *A Yang Universe* was not a casual fact; blue and red are yin and yang colors, clearly identified with Taoism and Buddhism, respectively. Yet, we could use color directives in just the opposite sense, to represent an arbitrary possibility within a continuum of possibilities, without any particular meaning, like in the visual metaphor of the *Primigenial Chaos* in Figure 7.

In Taoist cosmovision, the Heavenly Sage of Yuanshi (Xu 2009) was born before the origin of the universe and is the Lord of the first period of the universe, the *chaotic era*, in which the yin and yang had neither differentiated nor emerged yet. Everything was already in the Yuanshi era, yet undifferentiated and in a chaotic state. A randomly colored metaphor of the Yuanshi period is shown in Figure 7, designed and constructed using the same elementary algebraic tools described above. Formally speaking the *Primigenial Chaos* series was the first extended experiment with forms and colors, random effects, perception plays, and programming in Mathematica.

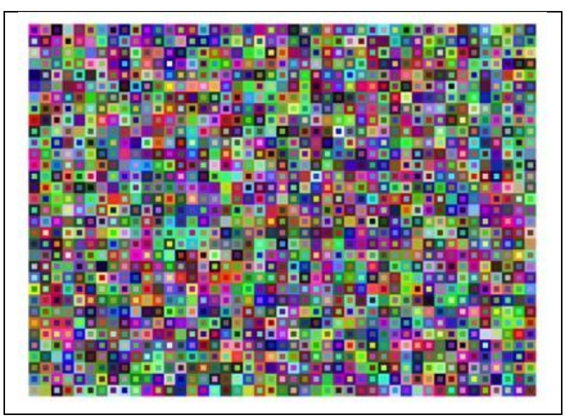

Figure 7. Primigenial Chaos. A metaphor of the Yuanshi period of the evolution of the Universe according to the Taoist cosmovision.

Figure 8 and Figure 9 show two images of the series *Nested Eccentric Chaos*, an afterwards evolution of the series *Primigenial Chaos*, designed to explore the effects the increasing of chaotic form and structure elements has on the subjective perception of chaos.

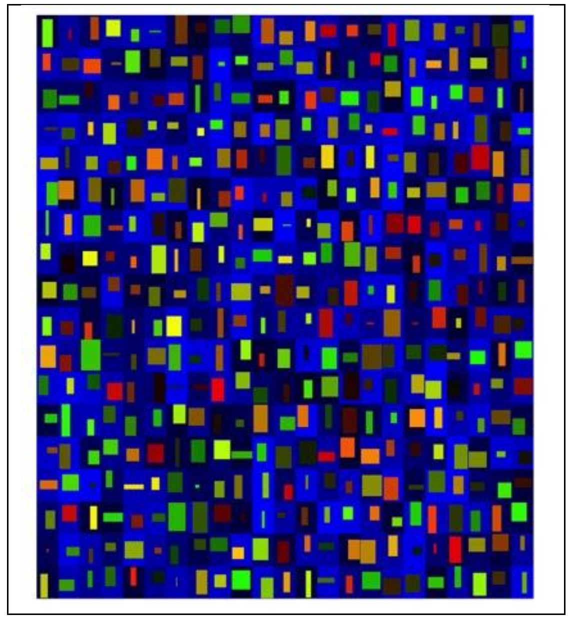

Figure 8. Nested Eccentric Chaos 1.

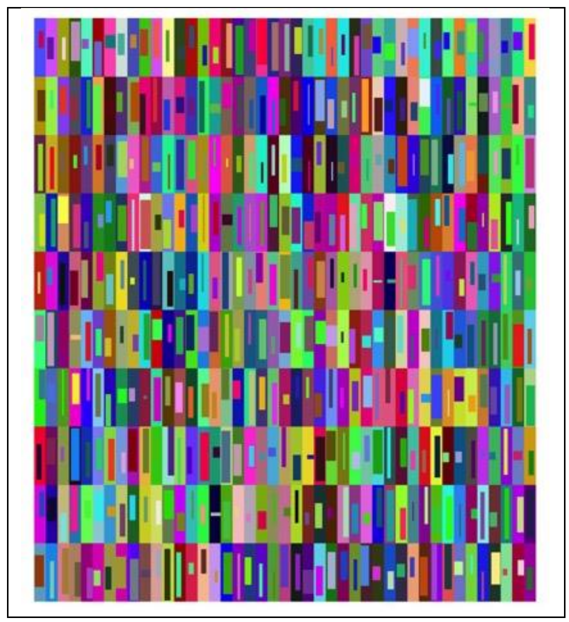

Figure 9. Nested Eccentric Chaos 2.

5.4 In the beginning was the Word

The existence of metaphors about the origin of the universe is a common feature of all cultures and civilizations around the world, which, very interestingly, permeates literature, music, and even contemporary science fiction. Perhaps the most widely spread Western metaphor for the origin of the universe is the biblical one, recorded in John 1 (KJV 1987):

“In the beginning was the Word, and the Word was with God, and the Word was God”.

We used John 1 as the starting inspiration point to construct multiple visual metaphors of the creation according to the Christian cosmovision. Figure 9 belongs to the series *In the Beginning was the Word*. Before the beginning there were only alphabets: the alphabets of mathematics, music, languages, chemical elements, etc.

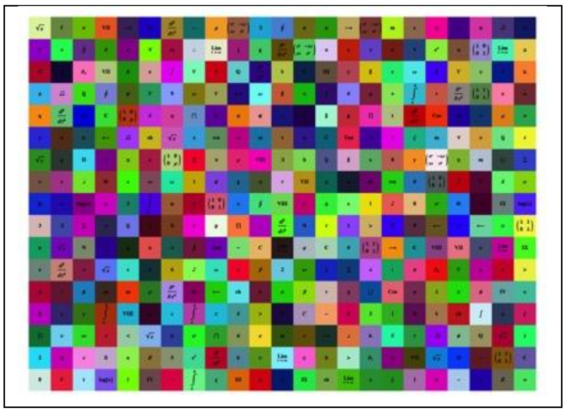

Figure 9. An image of the series *In the beginning was the word*.

5.5 Stained-Glass Windows

I love stained-glasses windows. Figure 10 shows an essay of designing and constructing an abstract digital stained-glass window, intended to play with the perception of luminosity through changes in the quotient of browns to reds as the observer reads the image in Western key.

5.6 Regular and Irregular Partitions

In this essay, we have deliberately maintained mathematics and coding at the lowest possible conceptual and technical level, and consistently have only considered regular rectangular partitions for the first layer of all described mosaics. Yet, this might not be so in more general cases. It is well known that the problem of partitioning plane surfaces using regular polygons has only three possible solutions: triangular, rectangular, and hexagonal partitions. In Figure 11, we show a very preliminary experiment of using triangular partitions to construct a landscape, whereas in Figure 12 we show the very first example of a double-layer hexagonal partition of a rectangular set of the real plane.

We close this gallery of images with Figure 13, another example of usage of the technical tools previously developed to construct a rather different kind of rectangular partition: a partition generated by a random walk (Wikipedia 2020), the kind of trajectories electrons describe in their movements across space.

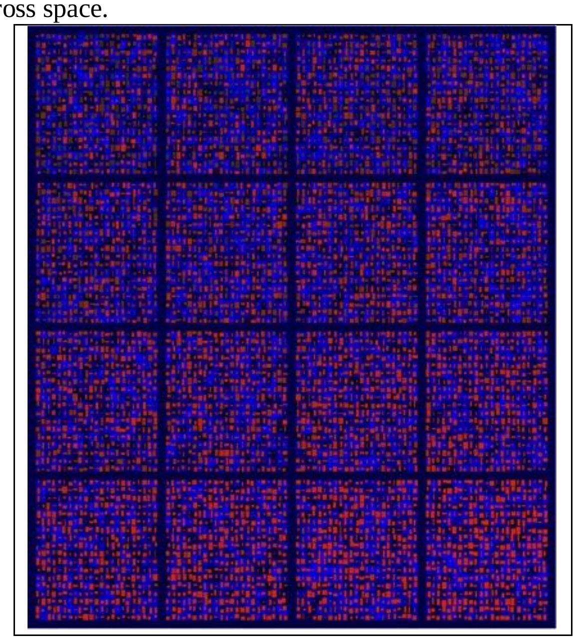

Figure 10. Perception Experiments. What we “see” depends on biology as much as it depends on cultural myths and patterns of behavior.

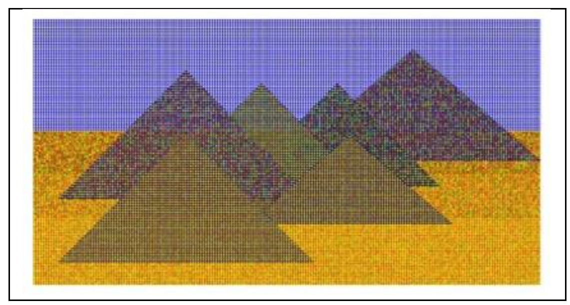

Figure 11. Experimental construction of a landscape using triangular partitions of the real plane.

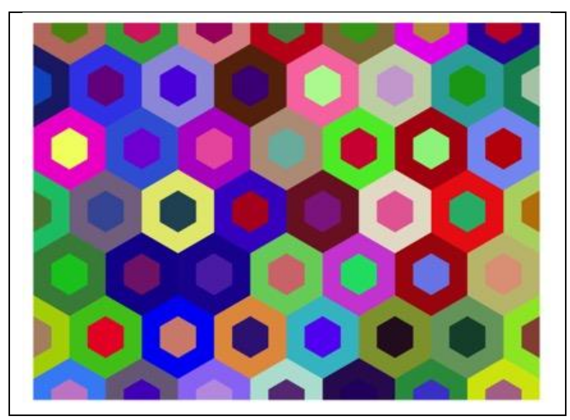

Figure 12. A double-layer hexagonal covering of a rectangular subset of the real plane.

Random walks are natural mathematical models for the unpredictable trajectories of processes like the saccadic movements of the eyes, the uncertain walk of a drunk, or the trajectories described by flies on the air. Yet, in what to the design and development of the next image is concerned, the inspiration came from imaging the Lord of the Yuanshi epoch searching for possible principles of topological organization to compartmentalize the universe in the middle of the primigenial chaotic era. It seems to be the Lord of the Yuanshi period found the score of this music too difficult to play at macroscopic levels, and then decided to leave it as a principle for the construction and implementation of local mechanisms at the microscopic scale.

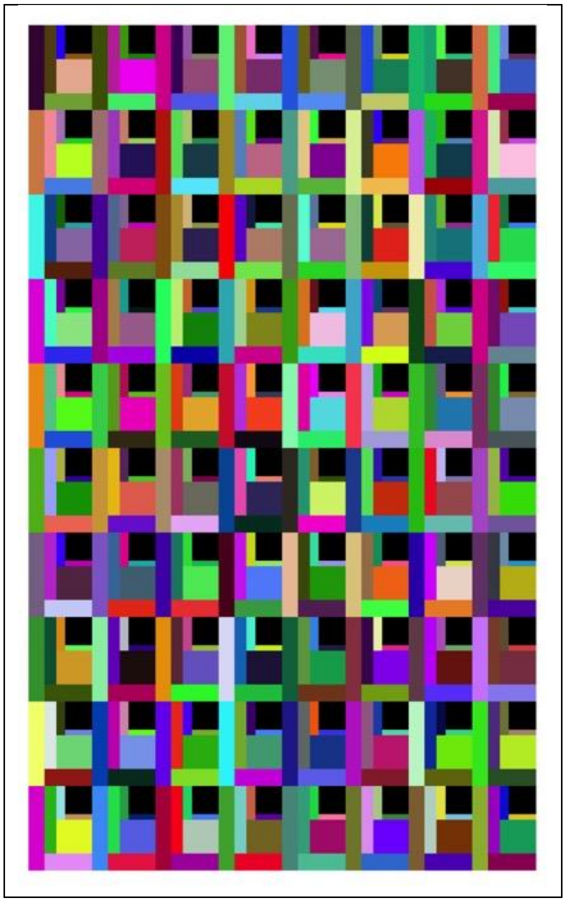

Figure 13. A stochastically colored partition generated by a random walk.

6 Discussion and Further Work

In spite of the importance and great relevance of the discussion about sciences and arts from the humanist point of view, in this paper we have left it aside, to focus our attention on exploring the possibility of pragmatic and productive complementary cooperation between geometry, algebra, dynamical systems, integrated symbolic-graphic-numerical computation, and visual arts, in the conception, development, and construction of artistic images. With this purpose in mind, and keeping the mathematical and computational technicalities to a strict minimum, we unveiled the secrets of designing a rectangular covering of a subset of the real plane, which would in principle allow drawing both abstract and figurative images with any desired resolution. Yet, like in any other field, quality and high resolution are always expensive, and so would it also be in this case, under the form of the computational power needed to compute long and complex codes in a reasonable time. Conceptually speaking the mathematical tool needed to support the whole process, the *method of exhaustion*, was already known to Archimedes around 2250 years ago (Apostol 1967). So, even though there is nothing new under the sun, the introduction of these elementary mathematical tools, together with a couple of elementary lectures on coding and programming, in the schools of visual arts would allow equipping visual art students with new and rich conceptual creative tools, capable of projecting them outside the set of merely blind users of precooked computation packages. On the other side of the spectrum, it would be highly desirable to offer sciences and engineering students the opportunity to incursion in the artistic fields, because arts, sciences, and engineering together permit approaching modeling complex systems from the complementary diverging perspectives and points of view so characteristic of visual and imagistic artistic thinking.

From the constructive technical point of view, this essay orbits around the concept of regular rectangular partition, to keep mathematics and programming requirements as low as possible. Triangular and hexagonal partitions, as in Figures 11 and 12, are technically more demanding. The programming supporting random rectangular partitions like in Figure 13 is still a little more involved. Yet, it is a big error to approach algorithmizable processes entirely from a programming point of view. Digital art is not an exception. Before start writing codes, it is of utmost importance to know and understand what one wants to do, because this is the only clever way to design as simple as possible, conceptually meaningful codes (Wolfram 2019).

During the last 40 years chaotic (Devaney 1989) and fractal (Barnsley 1988) dynamics, in association with graphical computation, have provided plenty of beautiful images reflecting the intrinsic beauty of nature and natural processes, but the supporting mathematical theory of dynamical systems behind it is completely out of the context of present work. Visual (Abraham-Shaw 1992) and artistic (Rodríguez-Millán 2020-1) exploitation of the images of sets of trajectories of dynamical systems, besides making them more attracting to the non-mathematically minded audience and being a rich source of beautiful inspiring images to abstract artists and designers, could also contribute to the construction of models and metaphors of complex concepts and systems because of their multicultural and intercultural nature, which might naturally promote unprejudiced syntheses arising from multiple disciplinary convergences and divergences. Suffice it to say at this point that an immense amount of work is still waiting to be done to bridge the gap between dynamical systems theory and audio-visual arts.

The history of mosaics as artistic objects goes back at least to the end of the 4^{th} millennia BC in Mesopotamia, and ran in continuous evolution until approximately the beginning of the Middle Age, all through the Middle East and Europe. The mosaics resurge in Rome in the XIX century and since then have been positively evolving anew until today (Visor 1999). Mathematically speaking mosaics define or realize partitions (Dugundji 1976) of plane surfaces, that is to say, a mosaic is a set of disjoint sets covering a certain subset of the real plane or, more generally, of a continuous bidimensional surface. So, all images in this essay are partitions of rectangular subsets of the real bidimensional plane. Partitions are the simplest forms of compartmentalizing sets and are omnipresent in everyday life, even if we do not perceive them consciously. Most existing academic institutions, and their study programs, for instance, are topological mosaics allowing the preservation of parcels of knowledge and traditions but impeding the interchange and cross-fertilization of technics and concepts. New kinds of topological structures are needed to promote interrelations and the appearance of systems with emergent properties. This will be a topic of work for years to come.

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