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Implicit self-tuning control for a class of nonlinear systems

Controlador auto-ajustable para una clase sistemas no lineales

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Abstract

The stability of implicit self-tuning control has been proved, for the discrete-time linear case, by the use of a Lyapunov function. Latter on the algorithm was extended for a class of bilinear systems. However real world systems are mostly nonlinear systems and it is of interest to extend the proposed algorithm to a more complex class of nonlinear models. In this research a nonlinear class of systems is defined, and then a generalized minimum variance control for the defined nonlinear class is developed. In addition, parameters of real world systems may change in time, and a good performance controller should be able to keep the overall system stability in such a case; to deal with this issue an implicit self-tuning control for the defined class of nonlinear systems is presented, the estimated parameters do not need to converge to their real values. The mathematical results show that with this new algorithm the self-tuning controller is able to keep the closed-loop system global stability for the defined class of nonlinear systems, and also the algorithm is a general case of the algorithms proposed in the literature for the bilinear and linear systems cases.

Palabras clave: Generalized minimum variance, nonlinear systems, self-tuning control, sliding mode control.

Resumen

La estabilidad de los controladores auto-ajustables ha sido demostrada, en el caso lineal discreto, usando una función de Lyapunov. Luego este algoritmo fue extendido a la clase de sistemas bilineales. Sin embargo, en el mundo real los sistemas en su mayoría son del tipo no lineales, por lo que es de gran interés extender el algoritmo propuesto a una clase más compleja de modelos no lineales. En esta investigación se define una clase de sistemas no lineales, y luego a esta clase se le desarrolla un controlador de mínima varianza generalizada. Además, en los sistemas reales los parámetros pueden cambiar en el tiempo, y un buen controlador debe ser capaz de lograr un buen desempeño y mantener la estabilidad global del sistema en lazo cerrado incluso en estos casos. Es por ello que se presenta un controlador auto-ajustable para tratar con las incertidumbres en los parámetros de la clase de sistemas no lineales ya definida, donde los parámetros estimados no necesariamente deben converger a los valores reales. Los resultados matemáticos demuestran que con este nuevo algoritmo el controlador auto-ajustable es capaz de mantener la estabilidad global del sistema en lazo cerrado, y además este algoritmo es un caso general que abarca los algoritmos antes presentados en la literatura para el caso de sistemas bilineales y lineales.

Palabras clave: Mínima varianza generalizada, sistemas no lineales, controlador auto-ajustable, control por régimen deslizante.

1 Introducción

One goal of control theory is to propose mathematical tools and algorithms to auto-regulate real process, given some desired specifications. Also one of the goals of several control theory researches is to find a control law that works for a large group of real process: linear or nonlinear, Single Input Single Output (SISO) or Multi Inputs Multi Outputs (MIMO), time invariant or time variant, and so on. However to find this type of controller is a hard work and is what keeps most of the control theory researches continuously working on it. On the other hand, the close-loop stability of controlled process is one of the most important issues to assure and prove when a new control algorithm is proposed.

The stability of implicit self-tuning control has been proved, for the linear discrete-time case, by the use of a Lyapunov function in (Patete y col., 2008a; 2008b), and for those systems, it suffices to use linear functions of the data to predict the system output response. The proposed algorithm was extended to the case where the linear discrete-time system is subject to white noise (Patete y col., 2008c), i.e. ARX (AutoRegresive with eXternal input) model. Several real systems have multi inputs multi outputs and, for that type of systems the results given in (Patete y col., 2008a; 2008b) were extended to the MIMO case in (Sugiki y col., 2008) and (Furuta y col., 2011).

It has been shown under relatively mild conditions that a large class of nonlinear systems can be approximated with arbitrary precision using bilinear models with finite number of coefficients. Bilinear systems are the simplest class of nonlinear systems and can also be regarded as a practical starting point for the study of other nonlinear systems. In addition, many concepts associated with linear systems can be extended to the bilinear case. A new algorithm was proposed, based on the results in (Patete y col., 2008a; 2008b), for the self-tuning control combining recursive parameters estimation and generalized minimum variance criterion, for a class of bilinear systems in (Patete y col., 2008d; 2011a), and also for an extended and more relaxed class of bilinear systems, where the control action could be presented only in the bilinear term in (Patete y col., 2010; 2014). However real world systems are mostly nonlinear systems and it is of interest to extend the proposed algorithm to a more complex class of nonlinear models. In general, it may be desirable, to consider the use of nonlinear functions to get good predictions and hence good control performance.

The paper is organized as follows: section 2 presents the problem to be solved; in section 3, the nonlinear system class to deal with is defined. Section 4 presents the generalized minimum variance criterion for the defined system class and, in section 5 the recursive implicit self-tuning algorithm based on the generalized minimum variance criterion is studied and, the main results are given by the theorem and proof which assure closed-loop system global stability. Some remarks conclude the paper.

2 The problem

Consider the general, Single Input Single Output (SISO), structure in the discrete-time case of a nonlinear system model as in (1),

$$A(z,q)y_k = B(z,q)u_k , \tag{1}$$

where y_k is the output signal of the process, u_k is the input signal, z denotes the time shift operator: $z^{-d}y_k = y_{k-d}$; $A(z,q)$ and $B(z,q)$ are polynomial of the form:

$$A(z,q) = 1 + a_1(q)z^{-1} + a_2(q)z^{-2} + ... + a_n(q)z^{-n},$$
$$B(z,q) = b_0(q) + b_1(q)z^{-1} + b_2(q)z^{-2} + ... + b_m(q)z^{-m},$$

q in the general case is a function of the input and output signal of the process as in (2),

$$q = h(y_k, u_k) . \tag{2}$$

Let's consider now the first order model general case,

$$y_{k+1} + a(q)\, y_k = b(q)\, u_k , \tag{3}$$

with $q = h(y_k)$. If $a(q)$ and $b(q)$ are constant values independents from the output signal y_k, i.e. $a(q) = a_0$ and $b(q) = b_0$, then the case is the same as for linear, first order, systems considered in (Patete, 2008a). If $a(q) = a_0$ and $b(q) = b_0 + b_1 y_k$, then the case is the same as for bilinear, first order, systems considered in (Patete y col., 2008d; 2011a; 2010; 2014).

From the above explanation, the first step to deal with this type of nonlinear systems structure is to define how to choose function $q = h(y_k)$ (or $q = h(y_k, u_k)$ in the general case), which is to said how to choose $a(q)$ and $b(q)$ for the first order case (3).

For example, for the first order system (3), if $a(q) = a_0 + a_1 y_k$ and $b(q) = b_0$, then (4),

$$y_{k+1} + a_0 y_k + a_1 y_k y_k = b_0\, u_k ,$$
$$y_{k+1} + a_0 y_k + a_1 y_k^2 = b_0\, u_k . \tag{4}$$

Or the case when $a(q) = a_0 + a_1 y_k$ and $b(q) = b_0 + b_1 y_k$, then (5):

$$y_{k+1} + a_0 y_k + a_1 y_k^2 = b_0 u_k + b_1 y_k u_k , \quad .. \tag{5}$$

and for these cases, (4) and (5), no results have been given.

3 Definition of the nonlinear class

Consider the general nonlinear system model structure as in (1), and q is defined as in (2), where $h(y_k, u_k)$ is any function (linear or nonlinear).

In this paper, to define the nonlinear class to deal with, the function $h(y_k, u_k)$ is restricted to be a function depending only of the output process data, i.e. $h(y_k)$, then q is as follows:

$$q = h(y_k) = h_{11} + h_{12} y_{k+n-1} + h_{13} y_{k+n-1}^2 + \ldots + h_{1n} y_{k+n-1}^{n-1}. \quad (6)$$

When (6) is substituted in (1), a nonlinear model with polynomial structure is obtained. For example consider a nominal model of a first order SISO time invariant nonlinear system as in (7),

$$y_{k+1} + a_0(q) y_k = b_0(q)\, u_k, \quad (7)$$

where the functions $a_0(q)$ and $b_0(q)$ are polynomial and depend only of the output process signal as shown in (8) and (9):

$$a_0(q) = a_{11} + a_{12} y_k + a_{13} y_k^2, \quad (8)$$

$$b_0(q) = b_0 + b_1 y_k + b_2 y_k^2. \quad (9)$$

Using (8) and (9) in (7), (10) is obtained,

$$A(z^{-1}) y_k + \Delta_{y_k^2}(z^{-1}) y_k^2 + \Delta_{y_k^3}(z^{-1}) y_k^3 = z^{-d} \cdot (B(z^{-1}) u_k + \Gamma_{y_k u_k}(z^{-1}) y_k u_k + \Gamma_{y_k^2 u_k}(z^{-1}) y_k^2 u_k), \quad (10)$$

where $d = 1$ and:

$$A(z^{-1}) = 1 + a_{11} z^{-1},$$
$$\Delta_{y_k^2}(z^{-1}) = a_{12} z^{-1},$$
$$\Delta_{y_k^3}(z^{-1}) = a_{13} z^{-1},$$
$$B(z^{-1}) = b_0,$$
$$\Gamma_{y_k u_k}(z^{-1}) = b_1,$$
$$\Gamma_{y_k^2 u_k}(z^{-1}) = b_2.$$

Note in (10) that the nonlinear terms are polynomial (depend on y_k^2, y_k^3) and there are bilinear terms (depend on $y_k u_k$ and $y_k^2 u_k$).

In general, the class of nonlinear systems is defined as a SISO time invariant model (11) with the following structure:

$$y_{k+n} + a_1(q) y_{k+n-1} + a_2(q) y_{k+n-2} + \ldots + a_n(q) y_k = b(q) u_k, \quad (11)$$

with q as in (6).

4 Generalized minimum variance control for the defined nonlinear class

In this section a generalized minimum variance control in (Åströmy col., 1989) (Chang y col., 1989), based on the concept of discrete-time sliding mode control (Furuta, 1990; 1993, Slotine, Li, 1991), is proposed for the defined class of nonlinear systems.

Consider de general nonlinear model (11), if:

$$\begin{aligned} a_1(q) &= a_{11} + a_{12} y_{k+n-1} + \ldots + a_{1n} y_{k+n-1}^{n-1}, \\ a_2(q) &= a_{21} + a_{22} y_{k+n-2} + \ldots + a_{2n} y_{k+n-2}^{n-1}, \\ &\vdots \\ a_n(q) &= a_{n1} + a_{n2} y_k + a_{n3} y_k^2 + \ldots + a_{nn} y_k^{n-1}, \\ b(q) &= b_0 + b_1 y_k + b_2 y_k^2 + \ldots + b_n y_k^n, \end{aligned} \quad (12)$$

then, substituting (12) in (11), writing the equation in the present time k and grouping terms, the following (13) is obtained

$$A(z^{-1}) y_k + \Delta_{y_k^2}(z^{-1}) y_k^2 + \Delta_{y_k^3}(z^{-1}) y_k^3 + \ldots + \Delta_{y_k^n}(z^{-1}) y_k^n = z^{-d} (B(z^{-1}) u_k + \Gamma_{y_k u_k}(z^{-1}) y_k u_k + \Gamma_{y_k^2 u_k}(z^{-1}) y_k^2 u_k + \ldots + \Gamma_{y_k^n u_k}(z^{-1}) y_k^n u_k), \quad (13)$$

where $d = n$ and:

$$A(z^{-1}) = 1 + a_{11} z^{-1} + \ldots + a_{n1} z^{-n},$$
$$\Delta_{y_k^2}(z^{-1}) = a_{12} z^{-1} + a_{22} z^{-2} + \ldots + a_{n2} z^{-n},$$
$$\Delta_{y_k^3}(z^{-1}) = a_{13} z^{-1} + a_{23} z^{-2} + \ldots + a_{n3} z^{-n},$$
$$\vdots$$
$$\Delta_{y_k^n}(z^{-1}) = a_{1n} z^{-1} + a_{2n} z^{-2} + \ldots + a_{nn} z^{-n},$$

$$B(z^{-1}) = b_0,$$
$$\Gamma_{y_k u_k}(z^{-1}) = b_1,$$
$$\Gamma_{y_k^2 u_k}(z^{-1}) = b_2,$$
$$\vdots$$
$$\Gamma_{y_k^n u_k}(z^{-1}) = b_n.$$

Assumptions 1:

1) There are no common factors in:

$$(A(z^{-1}), B(z^{-1})), (A(z^{-1}), \Delta_{y_k^2}(z^{-1})), \ldots,$$
$$(A(z^{-1}), \Delta_{y_k^n}(z^{-1})), (A(z^{-1}), \Gamma_{y_k u_k}(z^{-1})), \ldots,$$
$$(A(z^{-1}), \Gamma_{y_k^n u_k}(z^{-1})).$$

2) The order of the system (n, m) in (1) is known.

3) The time delay, d , is known.

4) To compute the nominal control law, the polynomials

$$A(z^{-1}), \Delta_{y_k^2}(z^{-1}) \ldots \Delta_{y_k^n}(z^{-1}), B(z^{-1}), \Gamma_{y_k u_k}(z^{-1}), \ldots,$$

$\Gamma_{y_k^n u_k}(z^{-1})$ are assumed to be known.

The control objective of the control law is to minimize the variance of the linear controlled sliding mode variable s_{k+d} , defined as (14):

$$s_{k+d} = C(z^{-1})(y_{k+d} - r_{k+d}) + Q(z^{-1})u_k, \tag{14}$$

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where polynomials $C(z^{-1})$ and $Q(z^{-1})$ are define as in (15) and (16) respectively,

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$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_n z^{-n}, \tag{15}$$

$$Q(z^{-1}) = q_0(1 - z^{-1}), \tag{16}$$

Remark 1: Polynomial $C(z^{-1})$ is designed to be Schur (i.e is designed by assigning all characteristic roots inside the unit disk of the z-plane for discrete-time systems) and polynomial $Q(z^{-1})$ must be designed as in (16) for the reference tracking to be assured (Patete 2008a; 2008b).

Polynomials $C(z^{-1})$ and $Q(z^{-1})$ are to be designed, so the error signal e_k , defined as (17):

$$e_k = y_k - r_k, \tag{17}$$

where r_k is the reference signal. The idea of proposing (14) defining the error signal as in (17) is based on the discrete-time sliding mode control (Furuta 1990; 1993).

To derive the nominal control law, general model (13) is multiplied by $E(z^{-1})$, then:

$$\begin{aligned} &E(z^{-1})A(z^{-1})y_k + E(z^{-1})\Delta_{y_k^2}(z^{-1})y_k^2 + \\ &E(z^{-1})\Delta_{y_k^3}(z^{-1})y_k^3 + \ldots + E(z^{-1})\Delta_{y_k^n}(z^{-1})y_k^n \\ &= z^{-d}(E(z^{-1})B(z^{-1})u_k + E(z^{-1})\Gamma_{y_k u_k}(z^{-1})y_k u_k \\ &+ E(z^{-1})\Gamma_{y_k^2 u_k}(z^{-1})y_k^2 u_k + \ldots + \\ &E(z^{-1})\Gamma_{y_k^n u_k}(z^{-1})y_k^n u_k), \end{aligned} \tag{18}$$

where $E(z^{-1})$ is a polynomial of the form:

$$E(z^{-1}) = e_0 + e_1 z^{-1} + \ldots + e_{d-1} z^{-d+1}.$$

Using the Diophantine equation (Patete 2008a; 2008b) (Chang y col., 1968):

$$C(z^{-1}) = A(z^{-1})E(z^{-1}) + z^{-d}F(z^{-1}), \tag{19}$$

where,

$$F(z^{-1}) = f_0 + f_1 z^{-1} + \ldots + f_{n-1} z^{-n+1},$$

equation (18) is rewritten as:

$$\begin{aligned} &C(z^{-1})y_{k+d} = F(z^{-1})y_k + P_{y_k^2}(z^{-1})y_{k+d}^2 + \\ &P_{y_k^3}(z^{-1})y_{k+d}^3 + \ldots + P_{y_k^n}(z^{-1})y_{k+d}^n + \\ &E(z^{-1})B(z^{-1})u_k + P_{y_k u_k}(z^{-1})y_k u_k + \\ &P_{y_k^2 u_k}(z^{-1})y_k^2 u_k + \ldots + P_{y_k^n u_k}(z^{-1})y_k^n u_k, \end{aligned} \tag{20}$$

where:

$$P_{y_k^2}(z^{-1}) = -E(z^{-1})\Delta_{y_k^2}(z^{-1}),$$
$$P_{y_k^3}(z^{-1}) = -E(z^{-1})\Delta_{y_k^3}(z^{-1}),$$
$$\vdots$$
$$P_{y_k^n}(z^{-1}) = -E(z^{-1})\Delta_{y_k^n}(z^{-1}),$$

$$P_{y_k u_k}(z^{-1}) = E(z^{-1})\Gamma_{y_k u_k}(z^{-1})$$
$$P_{y_k^2 u_k}(z^{-1}) = E(z^{-1})\Gamma_{y_k^2 u_k}(z^{-1}),$$
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$$\vdots$$
$$P_{y_k^n u_k}(z^{-1}) = E(z^{-1})\Gamma_{y_k^2 u_k}(z^{-1}).$$

Combining (20) and (14), the variable s_{k+d} is:

$$\begin{aligned} s_{k+d} &= F(z^{-1})y_k + P_{y_k^2}(z^{-1})y_{k+d}^2 + \\ &\dots + P_{y_k^n}(z^{-1})y_{k+d}^n + G(z^{-1})u_k + \\ &P_{y_k u_k}(z^{-1})y_k u_k + \dots + P_{y_k^n u_k}(z^{-1})y_k^n u_k \\ &- C(z^{-1})r_{k+d}, \end{aligned} \tag{21}$$

where $G(z^{-1}) = E(z^{-1})B(z^{-1}) + Q(z^{-1})$.

Then, the generalized minimum variance control input required to vanish s_{k+d} in (14) is given by:

$$u_k = - \frac{F(z^{-1})y_k + P_{y_k^2}(z^{-1})y_{k+d}^2 + \dots + P_{y_k^n}(z^{-1})y_{k+d}^n - C(z^{-1})r_{k+d}}{G(z^{-1}) + P_{y_k u_k}(z^{-1})y_k + \dots + P_{y_k^n u_k}(z^{-1})y_k^n} \tag{22}$$

5 Self-tuning control for the defined nonlinear system class

As it is known, parameters of real world process may not be accurately known or precise measured, or even worst parameters may change in time, and a good performance controller should be able to keep the overall system stability in such a case. System (13) is considered as a system with the same structure, however parametric uncertainties is taking into consideration in this section. For the implicit self-tuning controller, the parameter of the nominal control law (22) are estimated each sampled time, under the following assumptions,

Assumptions 2:
1) The order of the system (13) is known.
2) The time delay, d, is known.
3) Polynomial $C(z^{-1})$ is Schur.
4) Polynomial $Q(z^{-1})$ is designed as in (16).
5) The considered system with parametric uncertainties is in the class of systems which can be stabilized by the polynomials $Q(z^{-1})$ and $C(z^{-1})$ designed for the nominal system model (13) (Patete, 2008a; 2008b).
6) The reference signal r_k is bounded, i.e. $|r_k| < \delta$ for all k, where δ is a positive constant.

The closed-loop stability of self-tuning control for the defined nonlinear systems class, based on generalized minimum variance criterion, is given by the following recursive estimation equations:

$$\begin{aligned} \hat{\theta}_k &= \hat{\theta}_{k-1} + \\ &\frac{\Gamma_{k-1}\phi_{k-d}}{1 + \phi_{k-d}^T \Gamma_{k-1}\phi_{k-d}}[s_k + C(z^{-1})r_k - \phi_{k-d}^T \hat{\theta}_{k-1}], \end{aligned} \tag{23}$$

And

$$\Gamma_k = \Gamma_{k-1} - \frac{\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T \Gamma_{k-1}}{1 + \phi_{k-d}^T \Gamma_{k-1}\phi_{k-d}}, \tag{24}$$

where

$$\begin{aligned} \phi_k^T &= [y_k, \dots, y_{k-n+1}, y_k^2, \dots, y_{k-n-d+1}^2, \dots, \\ &y_k^n, \dots, y_{k-n-d+1}^n, u_k, \dots, u_{k-d+1}, y_k u_k, \dots, \\ &y_{k-d+1}u_{k-d+1}, y_k^2 u_k, \dots, y_{k-d+1}^2 u_{k-d+1}, \\ &y_k^n u_k, \dots, y_{k-d+1}^n u_{k-d+1}] \end{aligned} \tag{25}$$

is the vector containing measured output and control signal data,

$$\begin{aligned}\theta^T = [\, & f_0,\ldots, f_{n-1}, P_{y_k^2 0},\ldots, P_{y_k^2 n+d-1},\ldots,\\ & P_{y_k^n 0},\ldots, P_{y_k^n n+d-1}, g_0,\ldots, g_{d-1}, P_{y_k u_k 0},\ldots,\\ & P_{y_k u_k d-1},\ldots, P_{y_k^n u_k 0},\ldots, P_{y_k^n u_k d-1}]\end{aligned} \tag{26}$$

is the vector containing the controller parameters, and

$$\begin{aligned}\hat{\theta}^T = [\, & \hat{f}_0,\ldots, \hat{f}_{n-1}, \hat{P}_{y_k^2\, 0},\ldots, \hat{P}_{y_k^2\; n+d-1},\ldots,\\ & \hat{P}_{y_k^n 0},\ldots, \hat{P}_{y_k^n\; n+d-1}, \hat{g}_0,\ldots, \hat{g}_{d-1}, \hat{P}_{y_k u_k 0},\ldots,\\ & \hat{P}_{y_k u_k d-1}\;,\ldots, \hat{P}_{y_k^n u_k 0},\ldots, \hat{P}_{y_k^n u_k d-1}\;]\end{aligned} \tag{27}$$

is the estimate of θ.

The controller uses identified parameters as follows:

$$u_k = - \frac{\hat{F}(z^{-1})y_k + \hat{P}_{y_k^2}(z^{-1})y_{k+d}^2 + \ldots + \hat{P}_{y_k^n}(z^{-1})y_{k+d}^n - C(z^{-1})r_{k+d}}{\hat{G}(z^{-1}) + \hat{P}_{y_k u_k}(z^{-1})y_k + \ldots + \hat{P}_{y_k^n u_k}(z^{-1})y_k^n} \tag{28}$$

Theorem 1: Given a positive definite matrix Γ_0 and the initial parameters vector $\hat{\theta}_0$, if the estimate $\hat{\theta}_k$ of the controller (28) satisfies the recursive equations (23) and (24), under the set of Assumptions 2, then the close-loop system, combined by the self-tuning controller (28), (23) and (24) for the class of nonlinear system (13) with parametric uncertainties is global stable.

Proof: s_{k+d} is written as:

$$\begin{aligned}s_{k+d} = & \phi_k^T \tilde{\theta}_{k+d} + \hat{F}(z^{-1})y_k + \hat{P}_{y_k^2}(z^{-1})y_{k+d}^2 + \ldots\\ & + \hat{P}_{y_k^n}(z^{-1})y_{k+d}^n + \hat{G}(z^{-1})u_k + \hat{P}_{y_k u_k}(z^{-1})y_k u_k\\ & + \ldots + \hat{P}_{y_k^n u_k}(z^{-1})y_k^n u_k - C(z^{-1})r_{k+d}\end{aligned} \tag{29}$$

where $\tilde{\theta}_k = \theta - \hat{\theta}_k$.

Using the control law (28), (29) is rewritten as:

$$s_{k+d} = \phi_k^T \tilde{\theta}_{k-d}. \tag{30}$$

Consider the candidate Lyapunov function:

$$V_k = \frac{1}{2}s_k^2 + \frac{1}{2}\tilde{\theta}_k^T \Gamma_k^{-1} \tilde{\theta}_k. \tag{31}$$

The time difference of (31) is:

$$\Delta V_k = V_k - V_{k-1}, \tag{32}$$

$$\Delta V_k = \frac{1}{2}s_k^2 - \frac{1}{2}s_{k-1}^2 + \frac{1}{2}\tilde{\theta}_k^T \Gamma_k^{-1}\tilde{\theta}_k - \frac{1}{2}\tilde{\theta}_{k-1}^T \Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}, \tag{33}$$

$$\begin{aligned}\Delta V_k = & -\frac{1}{2}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right)^T \Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right) + \frac{1}{2}\tilde{\theta}_k^T\\ & .\left(\Gamma_k^{-1} + \Gamma_{k-1}^{-1}\right)\tilde{\theta}_k - \frac{1}{2}s_{k-1}^2 + \frac{1}{2}s_k^2 - \tilde{\theta}_k^T \Gamma_{k-1}^{-1}\tilde{\theta}_{k-1},\end{aligned} \tag{34}$$

$$\begin{aligned}\Delta V_k = & -\frac{1}{2}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right)^T \Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right) +\\ & s_k^2 - \frac{1}{2}s_k^2 - \frac{1}{2}s_{k-1}^2 - \tilde{\theta}_k^T \Gamma_{k-1}^{-1}\tilde{\theta}_{k-1} +\\ & \frac{1}{2}\tilde{\theta}_k^T\left(\Gamma_k^{-1} - \Gamma_{k-1}^{-1}\right)\tilde{\theta}_k + \tilde{\theta}_k^T \Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}.\end{aligned} \tag{35}$$

From (30), s_k is:

$$s_k^2 = \tilde{\theta}_k^T \phi_{k-d}\phi_{k-d}^T \tilde{\theta}_k. \tag{36}$$

Substituting (36) into (35), the following relation is derived:

$$\begin{aligned}\Delta V_k = & -\frac{1}{2}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right)^T \Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right)\\ & -\frac{1}{2}s_{k-1}^2 + \frac{1}{2}\tilde{\theta}_k^T\left(\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^T\right)\tilde{\theta}_k +\\ & \tilde{\theta}_k^T \Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T \tilde{\theta}_k\right).\end{aligned} \tag{37}$$

The term:

$$\frac{1}{2}\tilde{\theta}_k^T\left(\Gamma_k^{-1}-\Gamma_{k-1}^{-1}-\phi_{k-d}\phi_{k-d}^T\right)\tilde{\theta}_k$$

in (37) can be made equal to zero as follows:

$$\Gamma_k^{-1}-\Gamma_{k-1}^{-1}-\phi_{k-d}\phi_{k-d}^T=0, \tag{38}$$

$$\Gamma_k=\left(\Gamma_{k-1}^{-1}+\phi_{k-d}\phi_{k-d}^T\right)^{-1}, \tag{39}$$

$$\Gamma_k=\Gamma_{k-1}-\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\left(\Gamma_{k-1}+\phi_{k-d}\phi_{k-d}^T\right)^{-1}, \tag{40}$$

that yields (24) by the matrix inversion lemma (Åström y col., 1989).

The term:

$$\tilde{\theta}_k^T\Gamma_{k-1}^{-1}\left(\tilde{\theta}_k-\tilde{\theta}_{k-1}+\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k\right)$$

in (37) also can be made equal to zero as described below:

$$\tilde{\theta}_k-\tilde{\theta}_{k-1}+\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k=0, \tag{41}$$

$$\tilde{\theta}_k+\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k=\tilde{\theta}_{k-1}, \tag{42}$$

$$\begin{aligned}&\left(I+\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\right)\tilde{\theta}_k=\\&\quad\left(I+\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\right)\tilde{\theta}_{k-1}-\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_{k-1},\end{aligned} \tag{43}$$

$$\tilde{\theta}_k=\tilde{\theta}_{k-1}+\frac{\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\left(\theta-\hat{\theta}_{k-1}\right)}{\left(1+\phi_{k-d}\Gamma_{k-1}\phi_{k-d}^T\right)}. \tag{44}$$

From (21):

$$s_k=\phi_{k-d}^T\theta-C(z^{-1})r_k, \tag{45}$$

thus (23) is derived.

Using the recursive equations (23) and (24) in (37), for $k=1$, the following relation is obtained:

$$V_1-V_0=-\frac{1}{2}s_0^2-\frac{1}{2}\left(\tilde{\theta}_1-\tilde{\theta}_0\right)^T\Gamma_0^{-1}\left(\tilde{\theta}_1-\tilde{\theta}_0\right). \tag{46}$$

Initially $\tilde{\theta}_1-\tilde{\theta}_0\neq 0$, then $V_1-V_0<0$ which gives that $V_1<V_0$ in (46). For $k=2$,

$$V_2+\frac{1}{2}s_1^2+\frac{1}{2}\left(\tilde{\theta}_2-\tilde{\theta}_1\right)^T\Gamma_1^{-1}\left(\tilde{\theta}_2-\tilde{\theta}_1\right)=V_1<V_0. \tag{47}$$

For $k=3$,

$$V_3+\frac{1}{2}s_2^2+\frac{1}{2}\left(\tilde{\theta}_3-\tilde{\theta}_2\right)^T\Gamma_2^{-1}\left(\tilde{\theta}_3-\tilde{\theta}_2\right)=V_2, \tag{48}$$

using (47) and (48), the following is obtained:

$$\begin{aligned}&V_3+\frac{1}{2}s_2^2+\frac{1}{2}s_1^2+\frac{1}{2}\left(\tilde{\theta}_3-\tilde{\theta}_2\right)^T\Gamma_2^{-1}\left(\tilde{\theta}_3-\tilde{\theta}_2\right)+\\&\frac{1}{2}\left(\tilde{\theta}_2-\tilde{\theta}_1\right)^T\Gamma_1^{-1}\left(\tilde{\theta}_2-\tilde{\theta}_1\right)=V_1<V_0.\end{aligned} \tag{49}$$

Then, for $k=N$, where N is large, the following relation is derived:

$$\begin{aligned}&V_N+\frac{1}{2}\sum_{k=2}^{N}[s_{k-1}^2+\left(\tilde{\theta}_k-\tilde{\theta}_{k-1}\right)^T\Gamma_{k-1}^{-1}\left(\tilde{\theta}_k-\tilde{\theta}_{k-1}\right)]=\\&V_1<V_0,\end{aligned} \tag{50}$$

$$\begin{aligned}&V_N+\frac{1}{2}\sum_{k=2}^{N}[s_{k-1}^2+\left(\tilde{\theta}_k-\tilde{\theta}_{k-1}\right)^T\Gamma_{k-1}^{-1}\left(\tilde{\theta}_k-\tilde{\theta}_{k-1}\right)]<\\&V_0<\infty.\end{aligned} \tag{51}$$

For any $k=N\ (k>2)$, inequality (51) holds. Equation (51) implies that s_N and $\left(\hat{\theta}_N-\hat{\theta}_{N-1}\right)$ vanish as N approaches infinity, thus ΔV_k is negative semi-definite for all k and the generalized minimum variance is minimized, which proves the overall closed-loop system stability.

As a result from the above proof, ϕ_k^T is bounded. This means that:

$$y_k < \infty, y_{k-1} < \infty, ..., y_{k-n+1} < \infty,$$
$$y_k^2 < \infty, \dots, y_{k-n-d+1}^2 < \infty,$$
$$\vdots$$
$$y_k^n < \infty, \dots, y_{k-n-d+1}^n < \infty,$$
$$u_k < \infty, u_{k-1} < \infty, ..., u_{k-m-d+1} < \infty,$$
$$y_k u_k < \infty, ..., y_{k-\varsigma(d-1)} u_{k-\varsigma(d-1)} < \infty,$$
$$y_k^2 u_k < \infty, \dots, y_{k-d+1}^2 u_{k-d+1} < \infty,$$
$$\vdots$$
$$y_k^n u_k < \infty, \dots, y_{k-d+1}^n u_{k-d+1} < \infty,$$

are bounded for all k. Furthermore as $k \to \infty$, $s_k \to 0$ and $\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right) \to 0$, which means that $\hat{\theta}_k$ goes to a constant value (not necessary the real value θ).

The actual value y_k is shown to be bounded as follows:

Multiplying (14) by $B(z^{-1})$,

$$B(z^{-1})s_{k+d} = B(z^{-1})C(z^{-1})y_{k+d} - B(z^{-1})C(z^{-1})r_{k+d} + B(z^{-1})Q(z^{-1})u_k, \tag{52}$$

$$B(z^{-1})s_k = B(z^{-1})C(z^{-1})y_k - B(z^{-1})C(z^{-1})r_k + z^{-d}B(z^{-1})Q(z^{-1})u_k, \tag{53}$$

and using (13):

$$\begin{aligned} B(z^{-1})s_k &= B(z^{-1})C(z^{-1})y_k - B(z^{-1})C(z^{-1})r_k + \\ &A(z^{-1})Q(z^{-1})y_k + \Delta_{y_k^2}(z^{-1})Q(z^{-1})y_k^2 + \dots \\ &+ \Delta_{y_k^n}(z^{-1})Q(z^{-1})y_k^n - \\ &z^{-d}\Gamma_{y_k u_k}(z^{-1})Q(z^{-1})y_k u_k - \dots \\ &- z^{-d}\Gamma_{y_k^n u_k}(z^{-1})Q(z^{-1})y_k^n u_k). \end{aligned} \tag{54}$$

$$\begin{aligned} y_k &= \frac{B(z^{-1})}{T(z^{-1})}s_k + \frac{B(z^{-1})C(z^{-1})}{T(z^{-1})}r_k - \\ &\frac{\Delta_{y_k^2}(z^{-1})Q(z^{-1})}{T(z^{-1})}y_k^2 - \dots - \frac{\Delta_{y_k^n}(z^{-1})Q(z^{-1})}{T(z^{-1})}y_k^n \\ &+ \frac{\Gamma_{y_k u_k}(z^{-1})Q(z^{-1})}{T(z^{-1})}y_{k-d}u_{k-d} + \dots + \\ &\frac{\Gamma_{y_k^n u_k}(z^{-1})Q(z^{-1})y_k^n u_k}{T(z^{-1})}y_{k-d}u_{k-d}, \end{aligned} \tag{55}$$

where $T(z^{-1})$ is defined as:

$$T(z^{-1}) = C(z^{-1})B(z^{-1}) + A(z^{-1})Q(z^{-1}). \tag{56}$$

The signal s_k was proved to go to zero as $k \to \infty$. The signal r_k is assumed to be bounded for all k and the signal $y_{k-d}u_{k-d}$ was proved to be bounded from the boundeness of vector ϕ_k^T. From the set of Assumptions 2, number 5 means that the closed-loop characteristic polynomial, considering the described plant with parametric uncertainties, in (1), $T(z^{-1})$, is Schur. Thus, y_k in closed-loop is proved to be bounded. Furthermore, the error $e_k = y_k - r_k$ is bounded.

To reinforce this proof, we use the proof proposed in (Ohata y col., 2014), as follows:

In time k,

$$\begin{aligned} s_{k+d} &= \phi_k^T\tilde{\theta}_k + \hat{F}(z^{-1})y_k + \hat{P}_{y_k^2}(z^{-1})y_{k+d}^2 + \dots \\ &+ \hat{P}_{y_k^n}(z^{-1})y_{k+d}^n + \hat{G}(z^{-1})u_k + \hat{P}_{y_k u_k}(z^{-1})y_k u_k \\ &+ \dots + \hat{P}_{y_k^n u_k}(z^{-1})y_k^n u_k - C(z^{-1})r_{k+d}. \end{aligned} \tag{57}$$

Equation (57) is rewritten as in (54), where $\tilde{\theta}_k = \theta - \hat{\theta}_k$,

$$s_{k+d} = \phi_k^T\theta - C(z^{-1})r_{k+d}. \tag{58}$$

Defining $\overline{s}_k$ as:

$$\overline{s}_k = s_k + C(z^{-1})r_k - \phi_{k-d}^T\hat{\theta}_k = \phi_{k-d}^T\tilde{\theta}_k. \tag{59}$$

Using the new Lyapunov function:

$$V_k = \frac{1}{2}\bar{s}_k^2 + \frac{1}{2}\tilde{\theta}_k^T \Gamma_k^{-1}\tilde{\theta}_k. \qquad (60)$$

The time difference of (60) is:

$$\Delta V_k = V_k - V_{k-1}, \qquad (61)$$

$$\begin{aligned}\Delta V_k &= -\frac{1}{2}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right)^T \Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right)\\ &\quad -\frac{1}{2}\bar{s}_{k-1}^2 + \frac{1}{2}\tilde{\theta}_k^T\left(\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^T\right)\tilde{\theta}_k\\ &\quad + \tilde{\theta}_k^T \Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k\right).\end{aligned} \qquad (62)$$

If the terms:

$$\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^T = 0,$$

and

$$\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k = 0,$$

are satisfied, then:

$$\begin{aligned}\Delta V_k &= -\frac{1}{2}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right)^T \Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right)\\ &\quad -\frac{1}{2}\bar{s}_{k-1}^2 \le 0.\end{aligned} \qquad (63)$$

This means that $\bar{s}_k$ converge to zero and $\tilde{\theta}_k - \tilde{\theta}_{k-1}$ converge to zero as $k \to \infty$, which implies that $\left\|\theta_k\right\|$ and $\left\|\bar{s}_k\right\|$ are bounded. Then $\bar{s}_k$ approaches zero because of the following relation.

$$\left|\bar{s}_k - s_k\right| = \left|\phi_{k-d}^T\tilde{\theta}_k - \phi_{k-d}^T\tilde{\theta}_{k-d} - \phi_{k-d}^T\hat{\theta}_k + C(z^{-1})r_k\right|, \qquad (64)$$

$$\begin{aligned}\left|\bar{s}_k - s_k\right| &= \Big|\phi_{k-d}^T\left(\tilde{\theta}_k - \tilde{\theta}_{k-d}\right) - \hat{F}(z^{-1})y_k -\\ &\quad \hat{P}_{y_k^2}(z^{-1})y_{k+d}^2 - \ldots - \hat{P}_{y_k^n}(z^{-1})y_{k+d}^n -\\ &\quad \hat{G}(z^{-1})u_k - \hat{P}_{y_k u_k}(z^{-1})y_k u_k - \ldots -\\ &\quad \hat{P}_{y_k^n u_k}(z^{-1})y_k^n u_k + C(z^{-1})r_k\Big|.\end{aligned} \qquad (65)$$

Using (28) in (65), then,

$$\left|\bar{s}_k - s_k\right| = \left|\phi_{k-d}^T\left(\tilde{\theta}_k - \tilde{\theta}_{k-d}\right)\right|, \qquad (66)$$

$$\begin{aligned}\left|\bar{s}_k - s_k\right| &\le \left|\phi_{k-d}^T\left(\tilde{\theta}_k - \tilde{\theta}_{k-d}\right)\right| +\\ &\quad \left|\phi_{k-d}^T\left(\tilde{\theta}_{k-1} - \tilde{\theta}_{k-2}\right)\right| + \ldots +\\ &\quad \left|\phi_{k-d}^T\left(\tilde{\theta}_{k-d} - \tilde{\theta}_{k-d}\right)\right|,\end{aligned} \qquad (67)$$

$$\left|\bar{s}_k - s_k\right| \le \left\|\phi_{k-d}^T\right\| \sum_{j=0}^{d-1}\left\|\tilde{\theta}_{k-j} - \tilde{\theta}_{k-j-1}\right\|. \qquad (68)$$

Thus, the output y_k approaches the reference r_k as $k \to \infty$ because $C(z^{-1})$ is Schur. The global stability of the considered closed-loop system using the implicit self-tuning controller is proved. □

This algorithm represent a more general implicit (or also called direct) self-tuning control, based on the nominal generalized minimum variance control, using the discrete-time sliding mode control concept. The algorithm may be applied to linear SISO system model defined as in (Patete y col., 2008a; 2008b), bilinear and a more relaxed class of bilinear systems models given by (Patete y col., 2008d; 2011a; 2010; 2014), and for the defined nonlinear system class exposed in this paper.

The algorithm presented in this work may be also extended to linear ARX SISO system model presented in (Patete y col., 2008c), time-variant systems model given in (Patete y col., 2007; 2011b), and MIMO system model as in (Sugiki y col., 2008) (Furuta y col., 2011), combining the proof given in this paper and the proof given in each reference, respectively.

8 Conclusions

A more general implicit self-tuning control algorithm, based on the nominal generalized minimum variance control, using the discrete-time sliding mode control concept was presented. The algorithm may be applied to linear SISO system models, bilinear models, and for the defined nonlinear system class defined in this paper. The proposed self-tuning approach enables controller parameters to be estimated. The closed-loop global stability of the proposed implicit self-tuning control for the defined class of nonlinear systems was proved. Control stability and reference tracking are shown to be assured. The given algorithm is based on the idea of the discrete-time sliding mode control concept. As a future work, the proposed algorithm is to be applied to some nonlinear process model to show its performance.

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