Stochastic modeling of fluid displacement in porous media and its impact in oil recovery engineering

Rafael Rangel (rerangel@usb.ve) y Sergio Rojas[1] (srojas@usb.ve)
Departamento de Física, Universidad Simón Bolívar
Valle de Sartenejas, Baruta, Edo. Miranda, Venezuela.

Abstract

The displacement of a viscous fluid by another that preferentially wets a porous medium is modeled with the aim to simulate cooperative invasion processes found in experiments of immiscible wetting displacement. In our model we consider the non-local effects on it of the Laplacian pressure field and the capillary forces. This is achieved with Diffusion Limited Aggregation DLA-type Montecarlo computations that simulate both the hydrodynamic equations in the Darcy regime with a boundary condition for the pressure at the interface. The boundary condition contains two different types of disorder: the capillary term which constitutes an additive random disorder, and a term containing an effective random surface tension which couples to a curvature (it constitutes a multiplicative random term that carries non-local information of the whole pressure). This multiplicative random disorder together with the non-local coupling causes a short range scaling regime that reveals itself in a roughness exponent $\alpha \approx 0.80$. Additionally, we find a DLA-type scaling regime with a roughness exponent $\alpha \approx 0.60$ at the largest scales. Our results sheds the following strategy for oil recovery: in order to minimize trapping of oil in the fingers, oil has to be displaced by a liquid that preferentially wets the porous rock.

Key words: DLA simulation; wetting displacement; Monte Carlo simulation; Porous Media Flow; Unstable displacement.

Resumen

Presentamos un modelo del desplazamiento de un fluido por otro que preferencialmente moja un medio poroso mediante el cual pretendemos simular procesos de invasión cooperativa que se han encontrado en experimentos de desplazamiento inmicible por mojabilidad. Para ello consideramos efectos no locales que sobre el proceso de desplazamiento tienen el campo de presión Laplaciano y las fuerzas capilares. Ello se logra mediante simulaciones Monte Carlo del tipo difusión de agregado limite , en las que se similan las ecuaciones hidrodinámicas en el régimen de Darcy que incluye una ecuación de borde en la interfase, la cual contiene dos tipos de desorden: el término capilar el cual constituye un desorden aleatorio aditivo y un término que contiene una tensión superficial efectiva aleatoria que se acopla a la curvatura (ello constituye un término aleatorio multiplicativo que incorpora información no local de toda la presión). Es nuestra propuesta que el término de desorden aleatorio multiplicativo junto al acople no local son los responsables de un régimen de escalamiento en escalas cortas que se manifiesta a si mismo en un exponente abrupto $\alpha \approx 0{,}8$. Además, encontramos un régimen de escalamiento de naturaleza DLA con un exponente abrupto $\alpha \approx 0{,}60$ en las escalas mayores, mientras que a escalas intermedias $\alpha \approx 0{,}70$ lo cual corresponde a percolación invasiva con atrapamiento. Los resultados proveen una estrategia para el

[1] Corresponding author

recobro de petróleo: para minimizar el adedamiento del petróleo en una roca porosa el mismo debe ser desplazado por un fluido que preferencialmente moje la roca porosa o al menos que sea comparable con la tendencia de mojabilidad del petróleo y a la más baja rata de extracción posible. Debe existir una especie de correlación negativa para minimizar el atrapamiento del petróleo en el adedamiento.

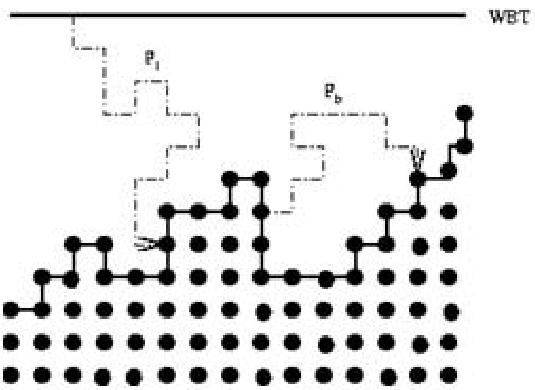
Key words: Simulación DLA; Desplazamiento por mojabilidad; Simulación de MonteCarlo; Flujo en medios porosos; Desplazamiento inestable.

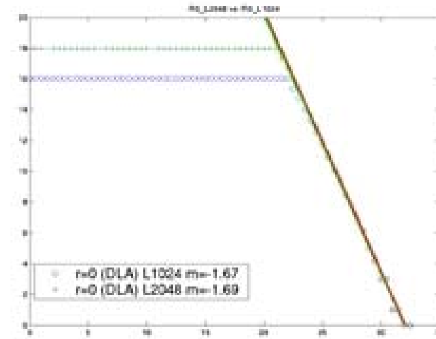
Introduction.- In the process of displacing a trapped fluid in porous media via the injection of an immiscible fluid, the phenomena of viscous fingering is one of the major reasons for inefficiency in secondary oil recovery. When in an oil field water is injected in order to displace oil, no more than half of the oil is pushed out of the field, due to the formation of fingers of water in oil, within which a good portion of oil remains trapped. Key to developing an strategy to optimize oil from secondary recovery (or efficiency of displacement) is an understanding of wetting displacement in porous media. That involves saturation of the injected fluid, its fingering and fractal character of the interface and the amount of trapped fluid. However, an understanding of wetting displacement in porous media has been elusive, and most of these efforts are in the direction of non-wetting displacement, i.e., when the displaced fluid wets the medium preferentially (Lovoll et al., 2004). In particular, in et al. (Ferer et al., 2004) experiments and pore network simulations concerning non-wetting invasiona have been done. Their experiments and simulations confirmed a prediction made some time ago (Fernandez et al., 1991; Rangel and Rivero, 1992) on the existence of a crossover length L_c separating the invasion percolation with trapping regime IPWT from the DLA-like regime, as well as its dependence on the capillary number $C_a = \mu U / \tau_{bar}$, where μ is the viscosity of the displaced fluid, U is the fluid velocity far from the interface, and τ_{bar} is the bare surface tension. Now, parameters involved in the description of immiscible fluid-fluid displacement in PM includes the viscosity ratio of the fluids (in both stable SD and unstable displacement UD), the permeability of the medium, the surface tension between the fluids, the relative ease with which the two fluids wet the medium together with the particular pore geometry(buoyancy forces are in our approach not considered). Experiments show that the relative ease of wetting by the displacing fluid defines the large scale shape of the displacement patterns. Relative wettability of fluids is relevant for understanding oil recovery and residual oil saturation (Tiab and Donaldson, 2004). In this context, Stokes et al.(Stokes et al., 1986) investigated the influence of wetting on unstable immiscible displacement (i.e. a very low viscosity μ_1 fluid displaces a much higher viscosity μ_2 one), finding that whenever the medium was preferentially wetted by the invading fluid (a process called wetting displacement WD), the width of the typical formed finger is always much larger than the pore size, following a scaling law that depends on the flow rate, the surface tension, and the permeability of the medium. On the other hand, if the displaced fluid wets the medium preferentially (non wetting displacement NWD), the experiments have shown that the width of the formed finger is of the order of the pore size. This is in qualitative agreement with earlier experimental patterns reported by Lenormand et al.(Lenormand and Zarcone, 1985), showing both IPWT-like patterns at low C_a and DLA-like patterns at high C_a (Fernandez et al., 1991; Rangel and Rivero, 1992),both in agreement with recent reported experiments and simulations (Ferer et al., 2004). Furthermore, Lenormad(Lenormand, 1990) investigated the physical mechanisms at the pore level for WD. He observed two processes: cooperative invasion of pores and collapse in throats when flow of the wetting fluid by film takes place (snapoff process). Which mechanism is dominant depends on the geometry of the PM. It basically depends on the relation of throat to pore sizes (small aspect ratio for similar sizes in a statistical sense and large aspect ratio otherwise), and possible on the viscosity ratio (as reported in recent model simulations (Al-Gharbi and Blunt, 2005)) implying that for high viscosity ratios the snapoff process is a rare event. More recently, Ganesan and Brenner (Ganesan and Brenner, 1998) intended an explicitly account of the nonlocal dynamics by combining the Laplace equation for the pressure p ($\nabla^2 p = 0$) with a boundary condition at the interface. The works in (Ganesan and Brenner, 1998) used the concept of a macroscopic interfacial surface tension. That is, they suppose the existence of an empirically effective surface tension τ_{eff} such that the pressure at the interface for small disturbances is given by the boundary condition

$$p = -\tau_{eff}\, \partial^2 h/\partial x^2 + \eta(x, h),$$

where h(x) represents the interface position and η(x, h) is a quenched disorder term representing the capillary forces, and the second derivative of h(x) approximates the gross curvature κ. The model we describe below makes use of the boundary condition Eq.(1) that uses a randomly distributed τ_{eff} that includes correlations with the capillary term. Our model is in contrast to pore-network simulators , an stochastic model.

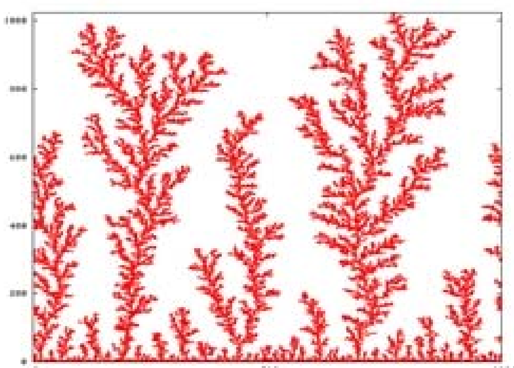
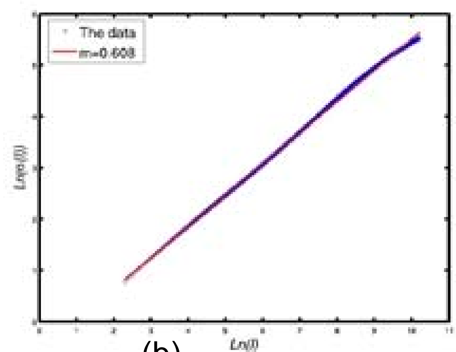
The Model. Specifically, we address the case of wetting invasion in PM of small aspect ratio and high viscosity ratio. Accordingly, we do not allow the invading fluid to break in compact disconnected clusters because of flow by film and snapoff processes, and only



(a)


(b)

Figure 1: (a) Walkers are released only from the Walkers Top Boundary (WTB) line on the viscous side with probability P_l due to the postulated vanishing viscosity of the displacing fluid; or from the interface boundary with probability P_b . Eq.(5) defines the ration of both probabilities and defines the Montecarlo scheme. (b) Mass box fractal dimension of the interface boundary of the DLA aggregate shown in fig.(2a.) for L=1024 and L=2048. Notice that there is not significance change in the scaling exponent $\simeq$ 1.70.

(a)



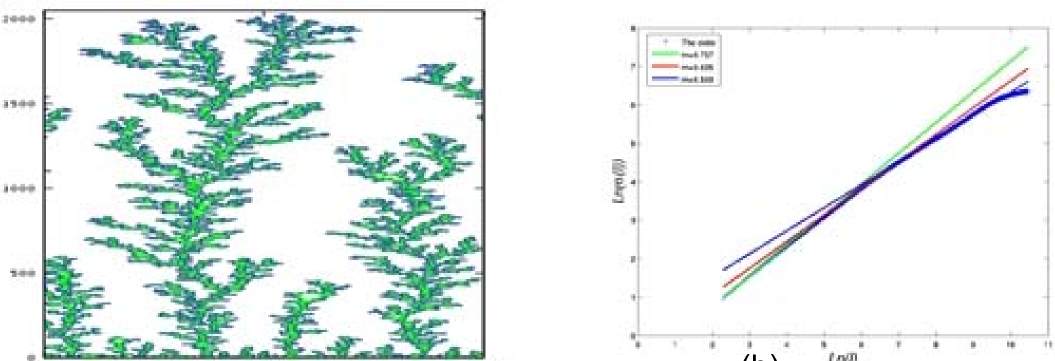
(b)

Figure 2: (a) The case r = 0 of eq.(5) defining the DLA aggregate and (b) its corresponding scaling function. The red line is the fit to the data (+) with a global Hurst exponent$\simeq$ 0.60. Notice the good scaling for around eight orders of magnitude

Wetting walkers are allowed. To model cooperative invasion we follow the idea that during the invasion process the menisci invades the pores cooperatively, after touching themselves by means of the surrounding throats, creating a single bigger meniscus inside the pore with bigger curvature (smaller pressure drop or the capillary pressure decreases) which is an unstable situation (due to an imbalance pressure from outside). This triggers the menisci movements until adjacent channels are invaded and the pressure balance is restored. Our model consider the non-local effect of viscous pressure drop via Laplacian growth combined with a boundary condition at the interface in the same scheme as (Fernandez et al., 1991; Rangel and Rivero, 1992). The boundary conditions aims to describe the cooperative process through the concept of correlation. We assume that the Laplace equation is fulfilled by the pressure at scales larger than the pore scale. Correspondingly, our model is intended to describe motion of the menisci at scales larger than the pore scale (Fernandez et al., 1991; Rangel and Rivero, 1992). Also, we assume the viscous fluid fulfills Darcy's law $\vec{v} = {k_m}/{\mu} \nabla p$, where $\vec{v}$ is the velocity field and κ_m is the medium permeability. If in addition the viscous fluid is assumed incompressible ($\nabla \cdot \vec{v} = 0$), one obtains $\nabla^2 p = 0$. This growth regime is realized via a Montecarlo DLA-type algorithm, which takes into account the drop pressure (or discontinuity of the pressure) across the interface (the general scheme of the Montecarlo algorithm is show in fig.(1a)). Since one contribution of this drop pressure is physically defined by the random capillary pressure in the throats, then in our simulations it is represented by a random function p_c (R), where R defines a site at the interface. The other contribution of the drop pressure at the interface has the form τ_{eff} (R)k(R), representing the pressure drop at the interface due to an effective random surface tension (here τ_{eff} (R) is a random function defined on each lattice site R, and k(R) is the curvature on it). A suitable notion of the curvature on a lattice is k = −δL/δA, where δL is the length increase of the interface and δA is the respective area swept as this length increases. Let's define

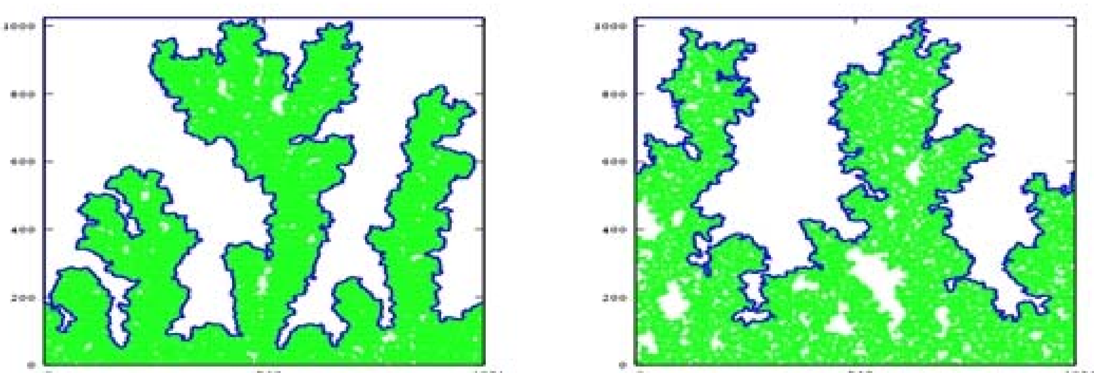
$$p' = p + \tau k_{\max} + \Delta p/2, \qquad (2)$$

where p_0 is both the pressure at the bottom of the cell and everywhere on the displacing fluid of negligible viscosity; Δp/2 is the half width of the dispersion of the numbers Δp_c, uniformly distributed between −Δp and +Δp; $(\tau_{eff}k)_{max}$ is the maximum value of the local curvature on the lattice times the greatest value of the function τ_{eff} (R). Clearly $\nabla^2 p' = 0$



(a) (b)

Figure 3: (a)Displacement pattern for q = 1 and r = 4, cell size L^2 = 2048 × 2048 and -Corr(the contour is in blue and the fingers are in green. Only a partial portion of the aggregate is shown in the horizontal direction). (b) Log-log plot of the scaling function σ(l),Eq. (7), of the aggregate shown in fig.1(a).



(a) (b)

Figure 4: Displacement pattern for q = 1 and r = 1024: (a) -Corr (b) +Corr. On each case the contour is in blue and the fingers are in green.

This definition of p' ensures that it will be positive everywhere, which enable us to interpret it as a probability. Because the pressure at the interface has to fulfill Eq. (1)

$$p(R) = \tau_{eff}(R)k(R) + p_c(R) + p_0 ,$$

we obtain on the viscous side of the interface that

$$p' = [(\tau k)_{max} - \tau_{eff}(R)k(R)) + (\Delta\bar{p}/2 - p_c(R)],$$

On the non-viscous side $p = p_0$. Since in Eq. (4) $\tau_{eff}(R)$ and $p_c(R)$ are random functions

defined on each lattice site R, we find it plausible and sufficient to take τ_{eff} from a uniform distribution in the interval [0,1]. Far away from the interface, on the viscous side, the pressure p decays to zero. Now ones forms the ratio between eq.(4) and eq.(2) evaluated far from the interface (p = 0), we finally arrive at the relation,

$$\frac{P_b(r)}{P_l} = r\left[q\left\{1 - \frac{2p(r)}{\Delta p}\right\} + \left\{1 + \frac{\tau(r)k(r)}{\tau k_{max}}\right\}\right], \qquad (5)$$

where $P_b(R)$ represents in the algorithm the probability of releasing a walker from the site R at the interface and P_l is the probability of releasing a walker from a line above the aggregate. Here,

$$r = \frac{\tau k_{max}}{a_0\langle\nabla p\rangle}, \qquad q = \frac{\Delta\tilde{p}/2}{\tau k_{max}}. \qquad (6)$$

where a_0 is the pore length scale which corresponds in our simulation to the lattice length scale and $\langle\Delta p\rangle = p_0/L$, the average macroscopic pressure drop along the cell system of length L.

Now, in order to model cooperative invasion in our Montecarlo simulations, the random functions $\tau_{eff}(R)$ and $p_c(R)$ have correlations between them as defined below. In Eq.(5), the parameter q is a measure for the relative importance of the capillary pressure at the pore level versus the cooperative process for the meniscus to advance. Using Darcy's law for the velocity $U = (\kappa_m/\mu)\langle\Delta p\rangle$, one obtains $r = (\kappa_m k_{max})/(a_0 C_a)$. The parameter $r \sim C_a^{-1}$ and it controls the growth of the cluster. The limit $r \to 0$ in eq.(5), defines the DLA limit or infinite high driving velocity U . This limit is shown in fig.(2a). On the other hand, the expressions inside brackets in Eq. (5) define rearrangement of the interface. This is the consequence of the existence of the effective surface tension. If the menisci at the interface are such that

the invading fluid form an obtuse angle (like mercury in glass), the cooperative process of the invasion is absent. Thus, in our picture expressed in Eq. (5) the first bracket vanishes, yielding the non-wetting limit (Ferer et al., 2004; Fernandez et al., 1991; Rangel and Rivero, 1992). The case when q =0 (i.e. the forces due to cooperative processes dominates over the capillary forces) implies having a disordered medium (the surface tension is random). The behavior for arbitrary (r, q) shows much more complexity. In Eq. (5) there are two kinds of statistical processes: the first pair of brackets represents a process in which the effective surface tension gross curvature give rise to the viscous fingering instability (fingers are boosted); the second pair of brackets represents the random capillary forces due to the assumed homogeneous disordered PM. Here we have both positive and negative values with equal statistical weight describing the process of advancing or retreating of the interface in the throats. This process tends to produce displacement aggregates topologically similar IPWT.

Correlations.- In small aspect ratio PM with NWD tendency, we model menisci movement as follows: in the throats the meniscus moves via the second term, i.e. wherever in the lattice capillary forces are statistically big, implying that the second term in Eq. (5) add a small probability to this process (this implies slow advancement of the meniscus in the smallest throats) the first term also should add a small contribution to the process, meaning that cooperative effects in the pores are rare events. For the thickest throats, however, small capillary forces lead to a measurable contribution of the second term, and due to a postulated higher tendency of neighboring menisci to touch, it also leads to a similar contribution of the first term. We call this case positive correlated +Corr. On the other hand, for PM with a WD tendency, the behavior is just the opposite. We expect for example that for the thickest throats a small contribution of the second term goes along with a big contribution of the first term. We have assumed that in places where the big throats connect pores, the cooperative process is easier. We name this case -Corr. Figure 3(a) shows the growth of a displacement pattern (aggregate) for r = 4, q = 1 and -Corr. The figure shows fingers in porous media and should be compared qualitatively with fig. 1(b) of (Stokes et al., 1986). The contour is traced, i.e. we “walk” along the interface and obtain the perimeter, a random function Y_i ,(Y_i is the vertical coordinate or growing direction of clusters) whose fluctuations are obtained via the contour function σ(l),

$$\sigma\left(l_0\right)=\left\langle\left(\frac{1}{l_0}\sum_{i=1}^{L}(Y_i-\bar{Y})^2\right)^{1/2}\right\rangle \quad , \qquad (7)$$

on which $\bar{Y}$ denotes the average of Y_i , in the interval l and $\langle\ \rangle_{av}$ denotes averaging over samples of length l. For the DLA case(r = 0 in our model) shown in fig.(2a), fig.(2b) show the scaling of the contour function for this case. It confirms that the perimeter function is a self-afffine function,i.e.,

$$Y(l) \simeq \lambda^{-H} Y(\lambda l), \qquad (8)$$

Specifically, the DLA case was studied in ref.((Babarasi and Vicsek, 1990)). They find H ≃ 0.59 ± 0.02 in accordance with our results. Furthermore, fig.(1b), show the scaling of the mass fractal dimension of the boundary, one obtains D_b ≈ 1.69, confirming the general relation,(Voss, 1986),

$$D_b = \frac{1}{H} \qquad (9)$$

Figure 3(b) shows the Log-log plot of three scaling regions of σ(l). For small length scales,

$\sigma(l) \approx l^{\alpha}$ with α ≈ 0.8. For the largest length scales, α ≈ 0.6, corresponding to the value obtained for DLA-limit (r = 0). In between, α ≈ 0.7, which corresponds to the value obtained for the growth of clusters via the rules of IPWT (α ≈ 0.72). The cluster growth with r = 4 and q = 1 but +Corr shows a tendency to more trapping with a similar scaling behavior. For small enough capillary number, i.e., bigger values of r, a markedly different behavior between -Corr and +Corr exist. For negative correlation, see fig. 4(a), fingers are well developed and a small amount of trapping is present. The regime α ≈ 0.8 exist for all length scales present as shown in 3(c), whereas for +Corr the aggregate, see fig. 4(b), shows a cross over behavior from α ≈ 0.8 at small length scales to α ≈ 0.6 at larger length scales. We see a gradual drifting to smaller scales of DLA scaling behavior as r increases for +Corr, while for negative correlation the regime α ≈ 0.8 gains larger scales. For increasing values of q and -Corr, there is a qualitative change. It appears that the viscous fingering instability weakens, and a clear tendency to a compact growth exist with plenty of trapping of the defending fluid (fig. 4(c) shows an example). The IPWT-like regime at larger length scales cross over the existent α ≈ 0.8 region at smaller length scales. The DLA like regime does not show itself in the range of scaling because it is limited by the total available length of the

Interface. Therefore, there are cross-over lengths, L_c separating different regimes of scaling, which are dependent on the parameters (r, q) and on the correlation type. On secondary oil recovery and related problems, we see that in order to minimize the amount of trapped oil within the fingers, the displacement should be of a wetting type and with an injected fluid such that, the effective surface tension dominates over the rate of extraction and capillary disorder.

References

[1] Al-Gharbi, M. S. and Blunt, M. J. (2005) Dynamic network modeling of two-phase drainage in porous media. Phys. Rev. E , 71, 016308:1-16.

[2] Babarasi, A. L. and Vicsek, T. (1990) Tracing a diffusion-limited aggregate: Self-affine versus self-similar scaling. Phys. Rev. A, 41, 6881- 6883.

[3] Ferer, M., Ji, C., Bromhal, G. S., Cook, J., Ahmadi, G., and Smith, D. H. (2004) Crossover from capillary fingering to viscous fingering for immiscible unstable flow: Experiment and modeling. Phys. Rev. E , 70, 016303:1-7.

[4] Fernandez, J. F., Rangel, R., and Rivero, J. (1991) Crossover length from invasion percolation to diffusion-limited aggregation in porous media. Phys. Rev. Lett., 67, 2958 - 2961.

[5] Ganesan, V. and Brenner, H. (1998) Dynamics of two-phase fluid interfaces in random porous media. Phys. Rev. Lett., 81, 578 - 581.

[6] Lenormand, R. and Zarcone, C. (1985) Invasion percolation in an etched network: Measurement of a fractal dimension. Phys. Rev. Lett., 54, 2226 - 2229.

[7] Lenormand, R. (1990) Liquids in porous media. J. Phys. Condens. Matter , 2, SA79-SA88.

[8] Lovoll, G., Meheust, Y., Toussaint, R., Schmittbuhl, J., and Maloy, K. J. (2004) Growth activity during fingering in a porous Hele-Shaw cell. Phys. Rev. E , 70, 026301:1-12.

[9] Rangel, R. and Rivero, J. (1992) Tracing interfaces in porous media. Physica A, 191, 253-257.

[10] Stokes, J. P., Weitz, D. A., Gollub, J. P., Dougherty, A., Robbins, M. O., Chaikin, P. M., and Lindsay, H. M. (1986) Interfacial stability of immiscible displacement in a porous medium. Phys. Rev. Lett., 57, 1718 - 1721.

[11] Tiab, D. and Donaldson, E. C. (2004) Petrophysics . Elsevier.

[12] Voss, R. F. (1986) Characterization and measurement of random fractals. Physica Scripta , T13, 27-32.